REAP 1.11

Documentation

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This is a more detailed documentation of the `REAP` add-on for Mathematica. We describe the functions which allow to calculate the evolution of the neutrino mass matrix in different models (SM, MSSM, 2HDM). Besides a function reference there is short HowTo on how you can build your own model.

Contents

1 Introduction

The `REAP` (**R**enormalization group **E**volution of **A**ngles and **P**hases) package is a Mathematica package to solve the renormalization group equations (RGE) of the quantities relevant for neutrino masses, for example the dimension 5 neutrino mass operator, the Yukawa matrices and the gauge couplings. So far, the β-functions for the standard model (SM), the minimal supersymmetric standard model (MSSM) and the two higgs doublet model with $\mathbb{Z}_2$ symmetry (2HDM) with and without right-handed neutrinos are implemented. Heavy degrees of freedom such as singlet neutrinos can be integrated out automatically at the correct mass thresholds which are determined by a fixed-point iteration. Thus the evolution is described by several effective theories. In addition all models are implemented with Dirac neutrinos. By means of the `MixingParameterTools` package, the calculated running of the neutrino mass matrix can be translated into the running of the mixing parameters and the mass eigenvalues.
If you would like to refer to REAP in a publication or talk, please cite the accompanying paper hep-ph/0501272.

2 Installation

2.1 Automatic Installation under UNIX/Linux

Execute REAP.installer and you are done.

```
sh REAP.installer
```

After the execution the Mathematica packages are copied to `~/.Mathematica/Applications/REAP` and the documentation and notebooks are in a subdirectory of the working directory, which is called REAP.
In addition, you have to install the package MixingParameterTools. There is also a script which installs both REAP and MPT at the same time: REAP_MPT.installer.

2.2 Semi-Automatic Installation under UNIX/Linux

Unpack the archive REAP.tar.gz.

```
tar -xvzf REAP.tar.gz
```

Then go to the directory REAPInstall and execute the script install.sh.

```
cd REAPInstall
sh ./install.sh
```

The script copies the Mathematica packages to `~/.Mathematica/Applications/REAP`. The documentation and some sample notebooks are placed in a new subdirectory of the working directory called REAP. Hence, the folder REAPInstall can be deleted now.
In addition, you have to install the package MixingParameterTools. REAP and MPT can be installed simultaneously by using the archive REAP_MPT.tar.gz. The procedure is completely analogous to the one described above.

2.3 Installation by Hand

In order to install the package(s) manually, unpack the archive REAP.tar.gz first. Under UNIX/Linux, type

```
tar -xvzf REAP.tar.gz
```

On Windows systems, a program like WinZip can be used. Then move the directory REAP from the folder REAPInstall to the directory where the Mathematica add-ons are located, e.g.

```
mv REAPInstall/REAP ~/.Mathematica/Applications/
```

under UNIX/Linux. Under Windows XP, the path to the add-on directory should be something like `Application Data\Mathematica\Applications`. The documentation and some sample notebooks can be found in REAPInstall/Doc/REAP/.
In addition, you have to install the package MixingParameterTools. To install both REAP and MPT at the same time, you can use the archive REAP_MPT.tar.gz. The procedure is the same as above (except that the installation directory is called REAP_MPTInstall now), supplemented by an analogous step for moving the MPT directory, e.g.

```
mv REAP_MPTInstall/MixingParameterTools ~/.Mathematica/Applications/
```

3 First Steps

The following simple example demonstrates how to calculate the RG evolution of the neutrino mass matrix in the MSSM extended by three heavy singlet neutrinos.

(1) The package corresponding to the model at the highest energy has to be loaded. All other packages needed in the course of the calculation are loaded automatically.

```
Needs["REAP‘RGEMSSM‘"]
```

Note that ‘ is the backquote, which is used in opening quotation marks, for example.

(2) Next, we specify that we would like to use the MSSM with singlet neutrinos:

```
RGEAdd["MSSM"]
```

(3) Now we have to provide the initial values. Here we use the default values of the package (see Sec. 5 for details) and a simple diagonal pattern for the neutrino Yukawa matrix.

```
RGESetInitial[2*10^16,RGEY\[Nu]->{{1,0,0},{0,0.5,0},{0,0,0.1}}]
```

(4) `RGESolve[low,high]` solves the RGEs between the energy scales low and high. The heavy singlets are integrated out automatically at their mass thresholds.

```
RGESolve[100,2*10^16]
```

(5) Using `RGEGetSolution[scale,`*`quantity`*`]` we can query the value of the quantity given in the second argument at the energy given in the first one. For example, this returns the mass matrix of the light neutrinos at 100 GeV:

```
MatrixForm[RGEGetSolution[100,RGEM\[Nu]]]
```

(6) To find the leptonic mass parameters, we use the function `MNSParameters[`m_ν`,`Y_e`]` (which also needs the Yukawa matrix of the charged leptons). The results are given in the order $\{\{\theta_{12}, \theta_{13}, \theta_{23}, \delta, \delta_e, \delta_\mu, \delta_\tau, \varphi_1, \varphi_2\}, \{m_1, m_2, m_3\}, \{y_e, y_\mu, y_\tau\}\}$.

```
MNSParameters[RGEGetSolution[100,RGEM\[Nu]],RGEGetSolution[100,RGEYe]]
```

(7) Finally, we can plot the running of the mixing angles:

```
Needs["Graphics`Graphics`"]
mNu[x_]:=RGEGetSolution[x,RGEM\[Nu]]
Ye[x_]:=RGEGetSolution[x,RGEYe]
\[Theta]12[x_]:=MNSParameters[mNu[x],Ye[x]][[1,1]]
\[Theta]13[x_]:=MNSParameters[mNu[x],Ye[x]][[1,2]]
\[Theta]23[x_]:=MNSParameters[mNu[x],Ye[x]][[1,3]]
LogLinearPlot[{\[Theta]12[x],\[Theta]13[x],\[Theta]23[x]},{x,100,2*10^16}]
```

To produce nicer plots, the notebook RGEPlots.nb, which is included in the package, can be used.

In a second run, let us try some more modifications of the defaults. For example, model parameters can be changed by including a command after step (2):

```
RGESetOptions["MSSM",RGEtan\[Beta]->20]
```

Furthermore, we set the SUSY breaking scale to 200 GeV and use the SM as an effective theory below this scale.

```
RGEAdd["SM",RGECutoff->200]
```

The initial values of the neutrino mass parameters can be changed by adding replacement rules in step (3). For instance, to set the GUT-scale value of θ_{13} to $6°$ and the Majorana phases to $50°$ and $120°$:

```
RGESetInitial[2*10^16,
  RGEY\[Nu]->{{1,0,0},{0,0.5,0},{0,0,0.1}},RGE\[Theta]13->6 Degree,
  RGE\[CurlyPhi]1->50 Degree,RGE\[CurlyPhi]2->120 Degree]
```

The results of the RG evolution with these parameters are now obtained by repeating the above steps (4)–(7).

4 Reference

4.1 Implementation details

`REAP` is divided in three parts. The main part is `RGESolver` which provides a standard interface between the different models and the user. Thus the user does not have to know anything about the implementation details of the different models besides the parameters of the models. The second part are the different models, like `RGESM`, `RGEMSSM`, ... which contain the model specific parts of the package. The third part is formed by some utility packages (`RGEUtilities`, `RGEParameters`, `RGEInitial`, `RGEFusaokaYukawa`, `RGESymbol`, `RGETakagi`) which provide several useful functions to the different models. In principle, a user only needs a limited set of functions of `RGESolver`.

4.2 `REAP`RGESolver``

The package distinguishes between two different kind of functions. On the one hand, there are functions which directly work with the supplied models. They are named `RGE*Model*`. On the other hand, there are functions dealing with the models which are used as an effective field theory (EFT), i.e. have been added by `RGEAddEFT`. These functions are named `RGE*EFT*`.
At the beginning, all models have to be loaded by `RGERegisterModel` in order to make them accessible through `RGESolver`. `RGERegisterModel` takes as argument different functions to communicate with the model. After all models have been registered which is done by the packages, the

models are contained in, the user has to specify, how his sequence of EFTs is made up. Different models can be added as EFT by `RGEAddEFT`. The cutoff is specified by the option `RGECutoff`. Next, the initial values have to be supplied by the function `RGESetInitial`. Then the renormalization group equations are solved by executing `RGESolve` which uses `NDSolve` to numerically integrate the differential equations. Finally, the parameters can be obtained through `RGEGetSolution` at any scale. In order to illustrate the use of `REAP`, an example is given in Sec. 3 and the algorithm to solve the different ranges is demonstrated in the following example.
The setup is the MSSM extended by 3 right-handed neutrinos at the GUT scale of $2 \cdot 10^{16}$ GeV and set the SUSY breaking scale to 1 TeV. The initial values are set to the suggested values which are specified in Sec. 5. At first, we define the model and set the initial values.

```
RGEAddModel["MSSM"];
RGEAddModel["SM",RGECutoff->1000];
RGESetInitial[2 10^16];
```

The execution of `RGESolve[`$91.19, 2 \cdot 10^{16}$`]` solves the RGE and finds the scales where the right-handed neutrinos are integrated out.

(1) Solve the RGEs for the MSSM with 3 right-handed neutrinos between the GUT scale and the SUSY breaking scale without considering any thresholds.

(2) Find the heaviest right-handed neutrino with mass M_3 and add a new EFT by `RGEAddEFT["MSSM",RGECutoff->`M_3`, RGEIntegratedOut->1]`.

(3) Calculate initial values for MSSM with 2 right-handed neutrinos by matching κ, Y_ν, M and the other parameters at the scale where the first right-handed neutrino is integrated out.

(4) Solve the RGEs for the MSSM with 2 right-handed neutrinos between M_3 and the SUSY breaking scale.

(5) Find the second to heaviest right-handed neutrino with mass M_2 and add a new EFT by `RGEAddEFT["MSSM", RGECutoff->`M_2`, RGEIntegratedOut->2]`.

(6) Calculate initial values for MSSM with 1 right-handed neutrino.

(7) Solve the RGEs for the MSSM with 1 right-handed neutrinos between M_2 and the SUSY breaking scale.

(8) Find the lightest right-handed neutrino with mass M_1 and add a new EFT by `RGEAddEFT["MSSM0N", RGECutoff->`M_1`]`.

(9) Calculate initial values for MSSM without right-handed neutrinos.

(10) Solve the RGEs for the MSSM without right-handed neutrinos between M_1 and the SUSY breaking scale.

(11) Calculate initial values for the SM

(12) Since all right-handed neutrinos have been integrated out already, change SM to SM0N.

(13) Solve the RGEs for SM0N between the SUSY breaking scale and the mass of Z^0.

4.2.1 RGEAdd

`RGEAdd[model,`*`options`*`]` specifies that `model` should be used as an effective theory (EFT) up to a cutoff energy given in the *`options`*. If no cutoff is given, ∞ is used. *`options`* can also be used to specify various parameters such as $\tan\beta$. See Sec. 5 for a complete list of the models and options available.

```
RGEAdd["MSSM",RGEtan\[Beta]->50]
RGEAdd["SM",RGECutoff->10^3]
```

In this case, the MSSM with $\tan\beta = 50$ is used at high energies. Below 10^3 GeV (the SUSY breaking scale in this example), the SM is used as an EFT.

4.2.2 RGEAddEFT

This command is identical to `RGEAdd`.

4.2.3 RGEGetEFTOptions

Same as `RGEGetOptions`.

4.2.4 RGEGetInitial

`RGEGetInitial[]` returns the scale at which the initial values are given and the initial values.

4.2.5 RGEGetModelOptions

`RGEGetModelOptions[model name]` returns the options of the model `model name`.

```
RGEGetModelOptions["SM"]
```

4.2.6 RGEGetOptions

`RGEGetOptions[model]` returns the options set by `RGEAdd` or `RGESetOptions` for the EFT `model`. Wildcards can be used in `model`.

```
RGEGetOptions["SM*"]
```

This returns the options which are currently set for all EFTs whose names start with "SM".

4.2.7 RGEGetParameters

`RGEGetParameters[model]` returns the quantities that run in the `model`.

4.2.8 RGEGetSolution

`RGEGetSolution[scale,`*`parameter`*`]` returns the solution of the RGEs at the energy `scale`. The *`parameter`* (optional) specifies the quantity of interest (cf. Sec. 5 for the lists for each model). If no *`parameter`* is given, the values of all running quantities are returned.

```
RGEGetSolution[100,RGEM\[Nu]]
```

returns the neutrino mass matrix at 100 GeV.

```
RGEGetSolution[100]
```

returns all parameters at 100 GeV.

4.2.9 RGEGetTransitions

`RGEGetTransitions[]` returns the transitions (thresholds) between the various EFTs in a list containing the energy scale, the model name and its options.

4.2.10 RGELoadAll

`RGELoadAll[filename]` loads the saved state which is given in `filename`.

4.2.11 RGELoadResults

`RGELoadResults[model]` loads the saved model which is given in `model`.

4.2.12 RGERegisterModel

`RGERegisterModel[name, get parameters, solve RGE, return solution, transition functions, provide initial values, set options, get options]` of the package `REAP` registers a new model. Its 8 parameters are:

(1) a string containing the name of the model

(2) a string containing the name of the package

(3) a function returning a list of the parameters of the model

(4) a function to solve the RGE of this model in a given range

(5) a list of replacement rules with the functions to return the result, like *Symbol→function returning the solution*

(6) a list containing the transition functions in the form {*"name of the target model",name of the function*}

(7) a function to provide initial values

(8) a function to set options of the model

(9) a function to get the initial values

```
RGERegisterModel["SM","SolveNeutrinoRGES`RGESM`" `Private`GetParameters,
        `Private`SolveModel, {RGEAll->`Private`GetSolution,
        RGEM\[Nu]->`Private`GetM\[Nu], RGEMe->`Private`GetMe,
        RGEMu->`Private`GetMu, RGEMd->`Private`GetMd},
        {{"SM",`Private`TransSM},{"SM0N",`Private`TransSM0N}},
        `Private`GetInitial, `Private`ModelSetOptions, `Private`ModelGetOptions
        ];
```

This example registers the SM. There are 5 functions to return solutions: Private‘GetSolution, Private‘GetMν,. . . . Moreover the only 2 transition functions are the transition functions to the SM (with righthanded neutrinos) itself: {"SM",Private‘TransSM} and to the SM without righthanded Neutrinos {"SM0N",Private‘TransSM0N}.

4.2.13 RGEReset

`RGEReset[]` removes all EFTs and resets all options which have been changed by `RGEAdd` or `RGESetOptions` to their default values. Options which have been changed by `RGESetModelOptions` are not reset.

4.2.14 RGESaveAll

`RGESaveAll[filename]` saves the state to `filename`.

4.2.15 RGESaveInitialData

`RGESaveInitialData[]` returns all data which is relevant to rerun the calculation,i.e. Initial values and the range. The data is returned as a list of replacement rules which is self-explaining (RGEUpperBound, RGELowerBound determine the range which is passed to `RGESolve`, RGEBoundaryScale is the scale where the initial data is given and RGEModelData is the model with cutoff and options. The remaining parameters are the initial values.).

4.2.16 RGESaveResults

`RGESaveModel[]` returns the current model. It can be loaded again by `RGELoadResults`.

4.2.17 RGESetEFTOptions

Same as `RGESetOptions`.

4.2.18 RGESetInitial

`RGESetInitial[scale,initial conditions]` sets the initial values at the energy `scale`. They are entered as replacement rules and can also contain options (e.g. to select the neutrino mass hierarchy). See Sec. 5 for the names of the variables and options in the different models. The option `RGESuggestion` chooses between several sets of default values. If it is not given, the first set of default values is taken. In general, these are the default values at the GUT scale.

```
RGESetInitial[10^16,RGE\[Theta]13->4 Degree,RGEMlightest->0.1]
```

This sets the initial values at 10^{16} GeV. The mixing angle θ_{13} is set to 4°, and the mass of the lightest neutrino to 0.1 eV. For the other parameters, the default values are used.

4.2.19 RGESetModelOptions

`RGESetModelOptions[model name,options]` globally changes the options of `model name` to `options`. Metacharacters, like * and @, are allowed in the `model name`. `model name` is matched against all model names with `StringMatchQ`.

```
RGESetModelOptions["SM",RGEvEW->246];
```

This sets the option RGEvEW of the “SM” to 246. The other options are unchanged.

4.2.20 RGESetOptions

`RGESetOptions[model,options]` changes the options of the EFTs defined by `RGEAdd` with name matching `model` to `options`. Metacharacters, like * and @, are allowed in the name.

```
RGESetOptions["MSSM",RGEtan\[Beta]->40]
```

This sets $\tan\beta$ of the “MSSM” to 40. The EFT must have been added earlier by `RGEAdd["MSSM"]`. The other options are unchanged.

4.2.21 RGESolve

`RGESolve[low,high,`*`options`*`]` solves the RGEs between the energies `low` and `high`. It accepts the same options as `NDSolve`. In addition, the option `RGERemoveAutoGeneratedEntries` determines whether automatically generated EFTs (such as the MSSM with 2 singlet neutrinos, if one started with 3 singlets) are removed before solving the RGEs. The default value is “True”. If it is set to “False”, no EFT will be removed.

```
RGESolve[100,10^15]
```

This solves the RGEs between 100 GeV and 10^{15} GeV.

4.3 REAP`RGESymbol`

`RGESymbol` defines several symbols which are used in exception handling and as parameters in RGEGetSolution.

4.4 REAP`RGEInitial`

This package contains some functions for converting mass and mixing parameters into mass matrices. They are mainly intended for internal use by REAP, but may be helpful for the user in some occasions.

4.4.1 RGEGetDiracYν

RGEGetDiracYν[θ_{12}, θ_{13}, θ_{23}, δ, δ_e, δ_μ, δ_τ, φ_1, φ_2, Mlightest, Δm^2_{atm}, Δm^2_{sol}, mass hierarchy, vu] returns a suggestion for Yν in the case of Dirac neutrinos.

- The first 9 parameters specify the mixing matrix.
- The 10th parameter is the mass of the lightest neutrino.
- The 11th and 12th parameter are the mass squared differences of the atmospheric and solar neutrino oscillations respectively.
- The 13th parameter is the mass hierarchy. "i" means inverted and "r" or "n" means normal.
- The 14th parameter is the vev of the Higgs coupling to the neutrinos.

4.4.2 RGEGetM

RGEGetM[θ_{12},θ_{13}, θ_{23},δ,δ_e,δ_μ,δ_τ,φ_1,φ_2,Mlightest,Δm^2_{atm},Δm^2_{sol},mass hierarchy, vu,Y_ν] returns a suggestion for the mass matrix of the right-handed neutrinos.

- The first 9 parameters specify the mixing matrix.
- The 10th parameter is the mass of the lightest neutrino.
- The 11th and 12th parameter are the mass squared differences of the atmospheric and solar neutrino oscillations respectively.
- The 13th parameter is the mass hierarchy. "i" means inverted and "r" or "n" means normal.
- The 14th parameter the neutrino Yukawa coupling matrix
- The 15th parameter is the vev of the Higgs coupling to the neutrinos.

4.4.3 RGEGetYν

RGEGetYν[$(Y_\nu)_{33}$,ratio] returns a suggestion of the Yukawa matrix of ν at the GUT scale. The first parameter is the mass of the heaviest neutrino and the second parameter specifies the mass ratio between the neutrinos. The result is a diagonal hierarchical matrix.

4.4.4 RGEGetYd

RGEGetYd[y_1, y_2, y_3, θ_{12},θ_{13}, θ_{23},δ,δ_e,δ_μ,δ_τ,φ_1,φ_2] returns a suggestion for Yd.

- The first 3 parameters are the eigenvalues of Y_d.
- The next 9 parameters are the mixing parameters.

4.4.5 RGEGetYe

`RGEGetYe[Yukawa` τ`]` returns a suggestion for the Yukawa matrix of the charged leptons at the GUT scale. The parameter is the Yukawa coupling of the τ. The suggested matrix is diagonal.

4.4.6 RGEGetκ

`RGEGet`κ`[`θ_{12}`,`θ_{13}`,`θ_{23}`,`δ`,`δ_e`,`δ_μ`,`δ_τ`,`φ_1`,`φ_2`,Mlightest,`Δm^2_{atm}`,`Δm^2_{sol}`,mass hierarchy,vu]` returns a suggestion for κ, the coupling of the dimension 5 operator, in GeV^{-1}.

- The first 9 parameters specify the mixing matrix.
- The 10th parameter is the mass of the lightest neutrino.
- The 11th and 12th parameter are the mass squared differences of the atmospheric and solar neutrino oscillations, respectively.
- The 13th parameter is the mass hierarchy. "i" means inverted and "r" or "n" means normal.
- The 14th parameter is the vev of the Higgs coupling to the neutrinos.

4.5 REAP`RGEParameters`

`RGEParameters` contains measured parameters of the SM like mixing angles, masses and coupling constants.

4.5.1 RGEMass

`RGEMass[particle name]` returns the mass of the given particle.

```
RGEMass["t"]
```

returns 174, the mass of the top quark.

4.5.2 gMZ

`RGEgMZ[i]` returns the value of the coupling constant i at the mass of the Z boson.

```
RGEgMZ[3]
```

returns the coupling constant of QCD at mZ.

4.6 REAP`RGEUtilities`

This package contains some functions needed by `RGESM`,`RGEMSSM` and `2HDM`.

4.6.1 RGEGetNeutrinoMasses

`RGEGetNeutrinoMasses[MassHierarchy,`Δm^2_{atm}`,`Δm^2_{sol}`,Mlightest]` converts its arguments into a list containing the neutrino mass eigenvalues m_1, m_2, m_3.

4.6.2 RGEGetRightHandedνMasses

`RGEGetRightHanded`ν`Masses[Scale]` returns the right-handed neutrino masses in a list, which is ordered by increasing mass. `Scale` is the scale at which the returned right-handed neutrino masses are defined.

4.6.3 RGEIntegrateOutM

`RGEIntegrateOutM[M,to be integrated out]` removes the rows and columns which are given in `to be integrated out` in `M`.

```
M=RGEIntegratedOutM[M,2];
```

removes the last 2 rows and columns of M.

4.6.4 RGEIntegrateOutYν

`RGEIntegrateOutY`ν`[Y`ν`,to be integrated out]` removes the rows which are given in `to be integrated out` in Yν.

```
Y=RGEIntegratedOutY\[Nu][Y,2];
```

removes the last 2 rows of Y.

4.6.5 RGERotateM

`RGERotateM[M,Y]` changes the basis to the eigensystem of `M` and returns `M` and `Y` in that basis.

4.6.6 RGESearchTransitions

`RGESearchTransitions[Mass,LogScale,MaxLogScale,Min,`*`Options`*`]` returns a list of transitions which are found by integrating out degrees of freedom. `Mass` is a function returning the mass matrix at a given scale and `LogScale`, `MaxLogScale` and `MinLogScale` are the starting point, the maximum and the minimum respectively.
Options of `RGESearchTransitions` are

- RGEPrecision is the precision used by `RGESearchTransitions` (default: 6)
- RGEMaxNumberIterations is the maximum number of iterations in the algorthm used to find a transition.(default: 20)
- RGEThresholdFactor determines when degrees of freedom are integrated out: RGEThresholdFactor*Mass=Scale where degree of freedom is integrated out (default: 1)

4.6.7 RGETestMYν

`RGETestMY`ν`[scale,M,`Y_ν`,`κ`]` integrates out the degrees of freedom in M which are above scale and returns M,Yν,κ,number of degrees of freedom which are integrated out
This function is useful to test whether right-handed neutrinos are already too heavy to be excited and therefore should already be integrated out. It can be used to check your initial values. The package checks whether there are right-handed neutrinos which are heavier than the initial scale. These are automatically integrated out and a warning is given.

4.7 `REAP‘RGETakagi‘`

This package contains a function implementing the Takagi diagonalization, which was implemented by Vinzenz Maurer following the algorithm described in Ref. [1].

4.7.1 RGETakagiDecomposition

`RGETakagiDecomposition[M]` performs a Takagi decomposition of `M` and returns in a list the unitary matrix `u` and the diagonalised matrix `d`, i.e. `u`,`d` with `d`=`u`.M.paramu^T. This was implemented by Vinzenz Maurer following the algorithm described in arXiv:physics/0607103 [physics.comp-ph].

```
{u,d}=RGETakagiDecomposition[M];
```

5 Models

5.1 Standard Model (SM)

5.1.1 REAP‘RGESM‘

This package contains the Standard Model extended by an arbitrary number of right-handed neutrinos (SM) to 1 loop order. It is possible to automatically find transitions where heavy neutrinos are integrated out. However, quarks are not integrated out.

Options:

- RGEIntegratedOut is the number of right-handed neutrinos which are integrated out. (default: 0)
- RGESearchTransition enables/disables the automatic search for transitions, i.e. automatically integrating out right-handed neutrinos. (default: True)
- RGEThresholdFactor determines where heavy degrees of freedom are integrated out: RGEThresholdFactor*Mass=Scale where degree of freedom is integrated out. (default: 1)
- RGEλ sets the initial value of the quartic Higgs coupling λ which is used when changing from the MSSM to the SM at M_{SUSY}. (default: 0.5)
- RGEvEW is the vev of the Higgs at the electroweak scale in GeV (default: 246). It is treated as a constant, i.e. its running is not taken into account.

Options used by `RGESetInitial`:
If the default values of all parameters are used, the resulting parameters will be compatible to the experimental data at the Z boson mass. The number of right-handed neutrinos is given by the initial conditions. There is no need to specify the number of neutrinos somewhere else.

- RGEMνr is the mass matrix of the right-handed neutrinos. If this parameter is specified, it also determines the light neutrino mass matrix via the see-saw formula (together with RGEYν). Thus, RGEMassHierarchy, RGEMlightest, RGEΔm2atm, RGEΔm2sol, RGEφ1, RGEφ2, RGEδ, RGEδe, RGE$\delta\mu$, RGE$\delta\tau$, RGEθ12, RGEθ13, and RGEθ23 do not have any effect in this case.
- RGEMassHierarchy is the hierarchy of the neutrino masses; ”r” or ”n” means normal hierarchy, ”i” means inverted hierarchy (default: ”r”).
- RGEMlightest is the mass of the lightest neutrino in eV (default: $\mathcal{O}(0.01)$ eV). The default of RGEMlightest depends on the model. It is chosen in such a way, that the parameters are compatible with the experimental data.
- RGEYν is the neutrino Yukawa matrix in “RL convention”. This option overrides the built-in Yukawa matrix, i.e. RGEYν33 and RGEYνRatio do not have any effect. (default: RGEGetYν(RGEYν33, RGE$Y\nu$Ratio))
- RGEYν33 is the (3,3) entry in the neutrino Yukawa matrix at the GUT scale. The default value depends on the model and it is chosen in such a way, that it is compatible with the experimental data (default: $\mathcal{O}(1)$).
- RGEYνRatio determines the relative value of the neutrino Yukawa couplings. The default value depends on the model and it is chosen in such a way, that it is compatible with the experimental data (default: $\mathcal{O}(1)$).
- RGEYd is the Yukawa matrix of the down-type quarks. If this parameter is given, RGEyd, RGEys, RGEyb, RGEqφ1, RGEqφ2, RGEqδ, RGEqδe, RGEq$\delta\mu$, RGEq$\delta\tau$, RGEqθ12, RGEqθ13, and RGEqθ23 are ignored.

- RGEYe is the charged lepton Yukawa matrix. If this parameter is given, RGEye, RGEyμ and RGEyτ are ignored.
(default: RGEGetYe(0.8*Mass["τ"]*Sqrt[2]/RGEvd))

- RGEYu is the Yukawa matrix of the up-type quarks. If this parameter is given, RGEyu, RGEyc and RGEyt are ignored; it is recommended not to use RGEqφ1, RGEqφ2, RGEqδ, RGEqδe, RGEq$\delta\mu$, RGEq$\delta\tau$, RGEqθ12, RGEqθ13, and RGEqθ23 in this case, since they are not necessarily equal to the CKM mixing parameters.

- RGEΔm2atm is the atmospheric mass squared difference (default: $\mathcal{O}(10^{-3})\,\mathrm{eV}^2$). The default value depends on the model and it is chosen in such a way, that it is compatible with the experimental data.

- RGEΔm2sol is the solar mass squared difference (default: $\mathcal{O}(10^{-4})\,\mathrm{eV}^2$). The default value depends on the model and it is chosen in such a way, that it is compatible with the experimental data.

- RGEφ1 and RGEφ2 are the Majorana CP phases φ_1 and φ_2 in radians (default: 0).

- RGEδ is the Dirac CP phase δ in radians (default: 0).

- RGEδe, RGE$\delta\mu$ and RGE$\delta\tau$ are the unphysical phases δ_e, δ_μ and δ_τ (default: 0).

- RGEκ is the coupling of the dimension 5 neutrino mass operator.

- RGEλ is the quartic Higgs self-coupling (default: 0.5). We use the convention that the corresponding term in the Lagrangian is $-\frac{\lambda}{4}(\phi^\dagger\phi)^2$.

- RGEθ12, RGEθ13 and RGEθ23 are the angles θ_{12}, θ_{13} and θ_{23} of the MNS matrix in radians. (default: $\theta_{13} = 0$ and $\theta_{23} = \frac{\pi}{4}$). The default of θ_{12} depends on the model. It is chosen in such a way, that the parameters are compatible with the experimental data.

- RGEg RGEg is the coupling constants of SU(5)

- RGEg1, RGEg2 and RGEg3 are the coupling constants of $\mathrm{U(1)_Y}$, $\mathrm{SU(2)_L}$ and $\mathrm{SU(3)_C}$, respectively. GUT charge normalization is used for g_1.

- RGEm RGEm is the Higgs mass

- RGEqφ1 and RGEqφ2 are the unphysical phases φ_1 and φ_2 of the CKM matrix which correspond to the Majorana phases in the MNS matrix (default: 0).

- RGEqδ is the Dirac CP phase δ of the CKM matrix.

- RGEqδe, RGEq$\delta\mu$ and RGE$\delta\tau$ are the unphysical phases δ_e, δ_μ and δ_τ of the CKM matrix (default: 0).

- RGEqθ12, RGEqθ13 and RGEqθ23 are the angles of the CKM matrix.

- RGEyd, RGEys and RGEyb are the Yukawa coupling of the down-type quarks d, s and b.

- RGEye, RGEyμ and RGEyτ are the Yukawa couplings of the charged leptons e, μ and τ.

- RGEyu, RGEyc and RGEyt are the Yukawa couplings of the up-type quarks u, c and t.

Parameters accepted by `RGEGetSolution`:

- RGECoupling is used to get the coupling constants.

- RGEGWCondition returns the Gildener Weinberg condition.

- RGEGWConditions returns all Gildener Weinberg conditions.

- RGEM1Tilde returns the effective light-neutrino mass $\widetilde{m}_1 = \frac{\left(m_D m_D^\dagger\right)_{11}}{M_1} = \frac{(Y_\nu Y_\nu^\dagger)_{11} v^2}{2M_1}$ which is commonly used in thermal leptogenesis. $\widetilde{m}_1$ is given in eV.
- RGEMν is used to get the mass matrix of the left-handed neutrinos.
- RGEMνr is the mass matrix of the right-handed neutrinos.
- RGEMd is used to get the mass matrix of the down-type quarks.
- RGEMe is used to get the mass matrix of the charged leptons.
- RGEMixingParameters returns the mixing parameters in the leptonic sector as they are returned by MNSParameters: $\{\{\theta_{12}, \theta_{13}, \theta_{23}, \delta, \delta_e, \delta_\mu, \delta_\tau, \varphi_1, \varphi_2\}, \{y_1, y_2, y_3\}, \{y_e, y_\mu, y_\tau\}\}$
- RGEMu is used to get the mass matrix of the up-type quarks.
- RGEPoleMTop is used to get the pole mass of the top quark in the $\overline{\text{MS}}$ scheme. The pole mass term of the top quark is given by

  $$m_t^{\text{Pole}} = m_t(m_t) \cdot (1 + \frac{4\alpha_s}{3\pi}) \tag{5.1}$$

  to 1-loop order.
- RGERawMνr is used to get the raw mass matrix of the right-handed neutrinos.
- RGERaw is used to get the raw values of all parameters. A raw parameter is the internal representation of the parameter
- RGERawYΔ is used to get the Yukawa coupling matrix of the coupling to the Higgs triplet.
- RGERawYν is used to get the raw Yukawa coupling matrix of the neutrinos.
- RGEAll returns all parameters of the model.
- RGEVEVratio returns the squared ratio of v_R over the EW symmetry breaking scale.
- RGEVEVratios returns the squared ratio of v_R over the EW symmetry breaking scale.
- RGEYν is used to get the Yukawa coupling matrix of the neutrinos.
- RGEYd is used to get the Yukawa coupling matrix of the down-type quarks.
- RGEYe is used to get the Yukawa coupling matrix of the charged leptons.
- RGEYu is used to get the Yukawa coupling matrix of the up-type quarks.
- RGEα is used to get the fine structure constants.
- RGEϵ1 is used to get the CP asymmetry [2] for leptogenesis for $M_1 \ll M_2, M_3$,

  $$\epsilon_1 = \frac{3}{8\pi}\frac{M_1}{v^2}\frac{\sum_{f,g} \text{Im}\left[(Y_\nu)_{1f}\,(Y_\nu)_{1g}\,(m_\nu^*)_{fg}\right]}{\left(Y_\nu Y_\nu^\dagger\right)_{11}} \,. \tag{5.2}$$

  Eq. (5.2) also holds if there are additional contributions to the neutrino mass operator, as it is for example the case in the type II see-saw mechanism [3].
- RGEϵ1Max is used to get the upper bound [4] on the CP asymmetry for leptogenesis in the type I see-saw mechanism for $M_1 \ll M_2, M_3$,

  $$\epsilon_1^{\max} = \frac{3}{8\pi}\frac{M_1 m_3}{v^2}\left[1 - \frac{m_1}{m_3}\left(1 + \frac{m_3^2 - m_1^2}{\widetilde{m}_1}\right)^{\frac{1}{2}}\right] \,. \tag{5.3}$$

- RGEλ is used to get the quartic Higgs self coupling.

5.1.2 REAP`RGESM0N`

This package contains the Standard Model without any right-handed neutrinos (SM0N) to 1 loop order.

It has the same parameters and options as `RGESM`, with the following exceptions: The only missing options are RGEIntegratedOut, RGESearchTransition, RGEThresholdFactor, RGEPrecision and RGEMaxNumberIterations, which are used to control the process of integrating out. Besides, RGEMνr and RGEYν are no parameters of `RGESetInitial`, and RGEϵMax, RGEϵ, RGEM1Tilde, RGERawMνr and RGERawYν are not accepted as parameters by `RGEGetSolution`. `RGESetInitial` has an additional option: RGESuggestion can be used to choose between different sets of default values, "GUT" (default) and "MZ". They refer to typical parameter values at the GUT scale or at the Z mass, respectively.

5.1.3 REAP`RGESMDirac`

This package contains the Standard Model with Dirac Neutrinos to 1 loop order.

It has the same parameters and options as `RGESM`, with the following exceptions: The only missing options are RGEIntegratedOut, RGESearchTransition, RGEThresholdFactor, RGEPrecision and RGEMaxNumberIterations, which are used to control the process of integrating out. In addition RGEκ and RGEMνr are no parameters of `RGESetInitial` and RGEMixingParameters, RGEϵMax, RGEϵ, RGEM1Tilde, RGERawMνr, RGERawYν and RGEκ are not accepted as parameters by `RGEGetSolution`. `RGESetInitial` has an additional option: RGESuggestion can be used to choose between different sets of default values, "GUT" (default) and "MZ". They refer to typical parameter values at the GUT scale or at the Z mass, respectively.

5.2 Minimal Supersymmetric Standard Model (MSSM)

5.2.1 REAP`RGEMSSM`

This package contains the Minimal Supersymmetric Standard Model extended by an arbitrary number of right-handed neutrinos (MSSM) to 1 and 2 loop order.
It is possible to automatically find transitions where heavy neutrinos are integrated out. But neither quarks are integrated out nor MSSM thresholds are considered.
Options:

- RGEIntegratedOut is the number of right-handed neutrinos which are integrated out. (default: 0)
- RGEModelVariant is a switch to change between different versions, but there are only two versions right now: 1Loop and 2Loop (default: 1Loop).
- RGESearchTransition enables/disables the automatic search for transitions, i.e. automatically integrating out right-handed neutrinos. (default: True)
- RGEThresholdFactor determines where heavy degrees of freedom are integrated out: RGEThresholdFactor*Mass=Scale where degree of freedom is integrated out. (default: 1)
- RGEΓd parameterizes the finite supersymmetric threshold corrections

  $$Y_d^{\mathrm{SM}} = Y_d^{\mathrm{MSSM}}(1 + \mathrm{RGE}\Gamma\mathrm{d}) * \cos(\beta) \tag{5.4}$$

  in the basis, in which Y_u is diagonal and the left-handed mixing is entirely contained in Y_d. It is related to the notation in [5]

  $$\mathrm{RGE}\Gamma\mathrm{d} \equiv \epsilon(V_{CKM}\Gamma_D^\dagger V_{CKM}^\dagger + \Gamma_U^\dagger) \tag{5.5}$$

  with $\epsilon = \tan\beta/(16\pi^2)$ and $\Gamma_{U,D}$ defines as in Eq. (1) of Ref. [5].

- RGEΓe parameterizes the finite supersymmetric threshold corrections

$$Y_e^{\text{SM}} = Y_e^{\text{MSSM}}(1 + \text{RGE}\Gamma\text{e}) * \cos\beta \tag{5.6}$$

in the basis, in which the Weinberg operator κ is diagonal and the left-handed mixing is entirely contained in Y_e. It is defined in a similar way to RGEΓd.

- RGEtanβ is the value of $\tan\beta = \frac{v_u}{v_d}$, the ratio of the 2 Higgs vevs (default: 50).

- RGEvEW is the combination $v = \sqrt{v_u^2 + v_d^2}$ of the Higgs vevs at the electroweak scale in GeV (default: 246). The vevs are treated as constants, i.e. their running is not taken into account.

Options used by `RGESetInitial`:
If the default values of all parameters are used, the resulting parameters will be compatible to the experimental data at the Z boson mass. The number of right-handed neutrinos is given by the initial conditions. There is no need to specify the number of neutrinos somewhere else.

- RGEMνr is the mass matrix of the right-handed neutrinos. If this parameter is specified, it also determines the light neutrino mass matrix via the see-saw formula (together with RGEYν). Thus, RGEMassHierarchy, RGEMlightest, RGEΔm2atm, RGEΔm2sol, RGEφ1, RGEφ2, RGEδ, RGEδe, RGE$\delta\mu$, RGE$\delta\tau$, RGEθ12, RGEθ13, and RGEθ23 do not have any effect in this case.

- RGEMassHierarchy is the hierarchy of the neutrino masses; "r" or "n" means normal hierarchy, "i" means inverted hierarchy (default: "r").

- RGEMlightest is the mass of the lightest neutrino in eV (default: $\mathcal{O}(0.01)\,\text{eV}$). The default of RGEMlightest depends on the model. It is chosen in such a way, that the parameters are compatible with the experimental data.

- RGEYν is the neutrino Yukawa matrix in "RL convention". This option overrides the built-in Yukawa matrix, i.e. RGEYν33 and RGEYνRatio do not have any effect. (default: RGEGetYν(RGEYν33, RGE$Y\nu$Ratio))

- RGEYν33 is the (3,3) entry in the neutrino Yukawa matrix at the GUT scale. The default value depends on the model and it is chosen in such a way, that it is compatible with the experimental data (default: $\mathcal{O}(1)$).

- RGEYνRatio determines the relative value of the neutrino Yukawa couplings. The default value depends on the model and it is chosen in such a way, that it is compatible with the experimental data (default: $\mathcal{O}(1)$).

- RGEYd is the Yukawa matrix of the down-type quarks. If this parameter is given, RGEyd, RGEys, RGEyb, RGEqφ1, RGEqφ2, RGEqδ, RGEqδe, RGEq$\delta\mu$, RGEq$\delta\tau$, RGEqθ12, RGEqθ13, and RGEqθ23 are ignored.

- RGEYe is the charged lepton Yukawa matrix. If this parameter is given, RGEye, RGEyμ and RGEyτ are ignored.
(default: RGEGetYe(0.8*Mass["τ"]*Sqrt[2]/RGEvd))

- RGEYu is the Yukawa matrix of the up-type quarks. If this parameter is given, RGEyu, RGEyc and RGEyt are ignored; it is recommended not to use RGEqφ1, RGEqφ2, RGEqδ, RGEqδe, RGEq$\delta\mu$, RGEq$\delta\tau$, RGEqθ12, RGEqθ13, and RGEqθ23 in this case, since they are not necessarily equal to the CKM mixing parameters.

- RGEΔm2atm is the atmospheric mass squared difference (default: $\mathcal{O}(10^{-3})\,\text{eV}^2$). The default value depends on the model and it is chosen in such a way, that it is compatible with the experimental data.

- RGEΔm2sol is the solar mass squared difference (default: $\mathcal{O}(10^{-4})\,\mathrm{eV}^2$). The default value depends on the model and it is chosen in such a way, that it is compatible with the experimental data.
- RGEφ1 and RGEφ2 are the Majorana CP phases φ_1 and φ_2 in radians (default: 0).
- RGEδ is the Dirac CP phase δ in radians (default: 0).
- RGEδe, RGE$\delta\mu$ and RGE$\delta\tau$ are the unphysical phases δ_e, δ_μ and δ_τ (default: 0).
- RGEκ is the coupling of the dimension 5 neutrino mass operator.
- RGEθ12, RGEθ13 and RGEθ23 are the angles θ_{12}, θ_{13} and θ_{23} of the MNS matrix in radians. (default: $\theta_{13} = 0$ and $\theta_{23} = \frac{\pi}{4}$). The default of θ_{12} depends on the model. It is chosen in such a way, that the parameters are compatible with the experimental data.
- RGEg RGEg is the coupling constants of SU(5)
- RGEg1, RGEg2 and RGEg3 are the coupling constants of $\mathrm{U(1)_Y}$, $\mathrm{SU(2)_L}$ and $\mathrm{SU(3)_C}$, respectively. GUT charge normalization is used for g_1.
- RGEm RGEm is the Higgs mass
- RGEqφ1 and RGEqφ2 are the unphysical phases φ_1 and φ_2 of the CKM matrix which correspond to the Majorana phases in the MNS matrix (default: 0).
- RGEqδ is the Dirac CP phase δ of the CKM matrix.
- RGEqδe, RGEq$\delta\mu$ and RGE$\delta\tau$ are the unphysical phases δ_e, δ_μ and δ_τ of the CKM matrix (default: 0).
- RGEqθ12, RGEqθ13 and RGEqθ23 are the angles of the CKM matrix.
- RGEyd, RGEys and RGEyb are the Yukawa coupling of the down-type quarks d, s and b.
- RGEye, RGEyμ and RGEyτ are the Yukawa couplings of the charged leptons e, μ and τ.
- RGEyu, RGEyc and RGEyt are the Yukawa couplings of the up-type quarks u, c and t.

Parameters accepted by `RGEGetSolution`:

- RGECoupling is used to get the coupling constants.
- RGEGWCondition returns the Gildener Weinberg condition.
- RGEGWConditions returns all Gildener Weinberg conditions.
- RGEM1Tilde returns the effective light-neutrino mass $\widetilde{m}_1 = \frac{\left(m_D m_D^\dagger\right)_{11}}{M_1} = \frac{(Y_\nu Y_\nu^\dagger)_{11} v^2}{2M_1}$ which is commonly used in thermal leptogenesis. $\widetilde{m}_1$ is given in eV. v is the vev of the Higgs doublet which couples to the neutrinos.
- RGEMν is used to get the mass matrix of the left-handed neutrinos.
- RGEMνr is the mass matrix of the right-handed neutrinos.
- RGEMd is used to get the mass matrix of the down-type quarks.
- RGEMe is used to get the mass matrix of the charged leptons.
- RGEMixingParameters returns the mixing parameters in the leptonic sector as they are returned by MNSParameters: $\{\{\theta_{12}, \theta_{13}, \theta_{23}, \delta, \delta_e, \delta_\mu, \delta_\tau, \varphi_1, \varphi_2\}, \{y_1, y_2, y_3\}, \{y_e, y_\mu, y_\tau\}\}$
- RGEMu is used to get the mass matrix of the up-type quarks.

- RGEPoleMTop is used to get the pole mass of the top quark in the $\overline{\text{MS}}$ scheme. The pole mass term of the top quark is given by

$$m_t^{\text{Pole}} = m_t(m_t) \cdot (1 + \frac{4\alpha_s}{3\pi}) \tag{5.7}$$

  to 1-loop order.

- RGERawMνr is used to get the raw mass matrix of the right-handed neutrinos.
- RGERaw is used to get the raw values of all parameters. A raw parameter is the internal representation of the parameter
- RGERawYΔ is used to get the Yukawa coupling matrix of the coupling to the Higgs triplet.
- RGERawYν is used to get the raw Yukawa coupling matrix of the neutrinos.
- RGEAll returns all parameters of the model.
- RGEVEVratio returns the squared ratio of v_R over the EW symmetry breaking scale.
- RGEVEVratios returns the squared ratio of v_R over the EW symmetry breaking scale.
- RGEYν is used to get the Yukawa coupling matrix of the neutrinos.
- RGEYd is used to get the Yukawa coupling matrix of the down-type quarks.
- RGEYe is used to get the Yukawa coupling matrix of the charged leptons.
- RGEYu is used to get the Yukawa coupling matrix of the up-type quarks.
- RGEα is used to get the fine structure constants.
- RGEϵ1 is used to get the CP asymmetry [2] for leptogenesis for $M_1 \ll M_2, M_3$,

$$\epsilon_1 = \frac{3}{4\pi}\frac{M_1}{v^2}\frac{\sum_{f,g}\text{Im}\left[(Y_\nu)_{1f}\,(Y_\nu)_{1g}\,(m_\nu^*)_{fg}\right]}{\left(Y_\nu Y_\nu^\dagger\right)_{11}} \ . \tag{5.8}$$

  Eq. (5.8) also holds if there are additional contributions to the neutrino mass operator, as it is for example the case in the type II see-saw mechanism [3].

- RGEϵ1Max is used to get the upper bound [4] on the CP asymmetry for leptogenesis in the type I see-saw mechanism for $M_1 \ll M_2, M_3$,

$$\epsilon_1^{\max} = \frac{3}{4\pi}\frac{M_1 m_3}{v^2}\left[1 - \frac{m_1}{m_3}\left(1 + \frac{m_3^2 - m_1^2}{\widetilde{m}_1}\right)^{\frac{1}{2}}\right] \ . \tag{5.9}$$

- RGEκ is used to get κ.

5.2.2 `REAP‘RGEMSSMON‘`

This package contains the Minimal Supersymmetric Standard Model (MSSM) without any right-handed neutrinos to 1 and 2 loop order.

It has the same parameter and options as `RGEMSSM`. The only missing options are RGEIntegratedOut, RGESearchTransition, RGEThresholdFactor, RGEPrecision and RGEMaxNumberIterations, which are used to control the process of integrating out. In addition RGEMνr and RGEYν are no parameters of `RGESetInitial` and RGEϵMax, RGEϵ, RGEM1Tilde, RGERawMνr and RGERawYν are not accepted as parameters by `RGEGetSolution`.

5.2.3 REAP‘RGEMSSMDirac‘

This package contains the MSSM with Dirac Neutrinos to 1 loop order and 2 loop order.

It has the same parameter and options as `RGEMSSM`. The only missing options are RGEIntegratedOut, RGESearchTransition, RGEThresholdFactor, RGEPrecision and RGEMaxNumberIterations, which are used to control the process of integrating out. In addition RGEMνr and RGEκ are no parameter of `RGESetInitial` and RGEMixingParameters, RGEϵMax, RGEϵ, RGEM1Tilde, RGERawMνr, RGERawYν and RGEκ are not accepted as parameters by `RGEGetSolution`.

5.3 Two Higgs Doublet Model (2HDM)

5.3.1 REAP‘RGE2HDM‘

This package contains the Two Higgs Doublet Model (2HDM) with a $\mathbb{Z}_2$ symmetry extended by an arbitrary number of right-handed neutrinos. The charged leptons always couple to the first Higgs. In addition there are right-handed neutrinos. The β-functions are to 1 loop order. The vevs of the Higgs fields are $v_1 = \langle\phi_1\rangle$ and $v_2 = \langle\phi_2\rangle$. They obey $v^2 = v_1^2 + v_2^2$, $v_1 = v\cos\beta$ and $v_2 = v\sin\beta$, where v is the v.e.v. of the SM Higgs and β ($\tan\beta = \frac{v_2}{v_1}$, $\beta \in \left(0, \frac{\pi}{2}\right)$) is used to parametrize the Higgs vevs.
Thus there are 2 dimension 5 operators which give mass to the light neutrinos.

$$\mathcal{L}_\kappa^{(ii)} = \frac{1}{4}\kappa_{gf}^{(ii)}\overline{l_{Lc}^C}^g \epsilon^{cd}\phi_d^{(i)} l_{Lb}^f \epsilon^{ba}\phi_a^{(i)} + \text{h.c.} \qquad (i = 1\ or\ 2)$$

The Higgs potential is

$$\begin{aligned}\mathcal{L}_{2Higgs} &= -\frac{\lambda_1}{4}\left(\phi^{(1)\dagger}\phi^{(1)}\right)^2 - \frac{\lambda_2}{4}\left(\phi^{(2)\dagger}\phi^{(2)}\right)^2 \\ &\quad -\lambda_3\left(\phi^{(1)\dagger}\phi^{(1)}\right)\left(\phi^{(2)\dagger}\phi^{(2)}\right) - \lambda_4\left(\phi^{(1)\dagger}\phi^{(2)}\right)\left(\phi^{(2)\dagger}\phi^{(1)}\right) \\ &\quad -\left[\frac{\lambda_5}{4}\left(\phi^{(1)\dagger}\phi^{(2)}\right)^2 + \text{h.c.}\right]\end{aligned}$$

The charged leptons always couple to the first Higgs field and the coupling of the other fields to the Higgs fields is determined by RGEModelOptions.
It is possible to automatically find transitions where heavy neutrinos are integrated out. But no other particles are integrated out.
Options:

- RGEIntegratedOut is the number of right-handed neutrinos which are integrated out. (default: 0)
- RGESearchTransition enables/disables the automatic search for transitions, i.e. automatically integrating out right-handed neutrinos. (default: True)
- RGEThresholdFactor determines where heavy degrees of freedom are integrated out: RGEThresholdFactor*Mass=Scale where degree of freedom is integrated out. (default: 1)
- RGEλ1, RGEλ2, RGEλ3, RGEλ4 and RGEλ5 set the initial values of the couplings λ_i $i \in \{1, \ldots, 5\}$ in the Higgs potential which are used when changing from the MSSM to the 2HDM at M_{SUSY} (default: $\lambda_1 = \lambda_2 = 0.75$, $\lambda_3 = \lambda_4 = 0.2$, $\lambda_5 = 0.25$).
- RGEtanβ is the value of $\tan\beta = \frac{v_2}{v_1}$, the ratio of the 2 Higgs vevs (default: 50).
- RGEvEW is the combination $v = \sqrt{v_1^2 + v_2^2}$ of the Higgs vevs at the electroweak scale in GeV (default: 246). The vevs are treated as constants, i.e. their running is not taken into account.

- RGEzν is a list defining the Higgs the neutrinos are coupling to. If the n^{th} component is one, the Higgs couples to the neutrinos. If it is 0, it won't couple (default: $\{0,1\}$). The charged leptons always couple to the first Higgs.
- RGEzd is a list defining the Higgs the down-type quarks are coupling to. If the n^{th} component is one, the Higgs couples to the down-type quarks. If it is 0, it won't couple (default: $\{1,0\}$).
- RGEzu is a list defining the Higgs the up-type quarks are coupling to. If the n^{th} component is one, the Higgs couples to the up-type quarks. If it is 0, it won't couple (default: $\{0,1\}$).

Options used by `RGESetInitial`:
If the default values of all parameters are used, the resulting parameters will be compatible to the experimental data at the Z boson mass. The number of right-handed neutrinos is given by the initial conditions. There is no need to specify the number of neutrinos somewhere else.

- RGEMνr is the mass matrix of the right-handed neutrinos. If this parameter is specified, it also determines the light neutrino mass matrix via the see-saw formula (together with RGEYν). Thus, RGEMassHierarchy, RGEMlightest, RGEΔm2atm, RGEΔm2sol, RGEφ1, RGEφ2, RGEδ, RGEδe, RGE$\delta\mu$, RGE$\delta\tau$, RGEθ12, RGEθ13, and RGEθ23 do not have any effect in this case.
- RGEMassHierarchy is the hierarchy of the neutrino masses; "r" or "n" means normal hierarchy, "i" means inverted hierarchy (default: "r").
- RGEMlightest is the mass of the lightest neutrino in eV (default: $\mathcal{O}(0.01)\,\text{eV}$). The default of RGEMlightest depends on the model. It is chosen in such a way, that the parameters are compatible with the experimental data.
- RGEYν is the neutrino Yukawa matrix in "RL convention". This option overrides the built-in Yukawa matrix, i.e. RGEYν33 and RGEYνRatio do not have any effect. (default: RGEGetYν(RGEYν33, RGE$Y\nu$Ratio))
- RGEYν33 is the (3,3) entry in the neutrino Yukawa matrix at the GUT scale. The default value depends on the model and it is chosen in such a way, that it is compatible with the experimental data (default: $\mathcal{O}(1)$).
- RGEYνRatio determines the relative value of the neutrino Yukawa couplings. The default value depends on the model and it is chosen in such a way, that it is compatible with the experimental data (default: $\mathcal{O}(1)$).
- RGEYd is the Yukawa matrix of the down-type quarks. If this parameter is given, RGEyd, RGEys, RGEyb, RGEqφ1, RGEqφ2, RGEqδ, RGEqδe, RGEq$\delta\mu$, RGEq$\delta\tau$, RGEqθ12, RGEqθ13, and RGEqθ23 are ignored.
- RGEYe is the charged lepton Yukawa matrix. If this parameter is given, RGEye, RGEyμ and RGEyτ are ignored.
(default: RGEGetYe(0.8*Mass["τ"]*Sqrt[2]/RGEvd))
- RGEYu is the Yukawa matrix of the up-type quarks. If this parameter is given, RGEyu, RGEyc and RGEyt are ignored; it is recommended not to use RGEqφ1, RGEqφ2, RGEqδ, RGEqδe, RGEq$\delta\mu$, RGEq$\delta\tau$, RGEqθ12, RGEqθ13, and RGEqθ23 in this case, since they are not necessarily equal to the CKM mixing parameters.
- RGEΔm2atm is the atmospheric mass squared difference (default: $\mathcal{O}(10^{-3})\,\text{eV}^2$). The default value depends on the model and it is chosen in such a way, that it is compatible with the experimental data.
- RGEΔm2sol is the solar mass squared difference (default: $\mathcal{O}(10^{-4})\,\text{eV}^2$). The default value depends on the model and it is chosen in such a way, that it is compatible with the experimental data.

- RGEφ1 and RGEφ2 are the Majorana CP phases φ_1 and φ_2 in radians (default: 0).
- RGEδ is the Dirac CP phase δ in radians (default: 0).
- RGEδe, RGE$\delta\mu$ and RGE$\delta\tau$ are the unphysical phases δ_e, δ_μ and δ_τ (default: 0).
- RGEκ1 is the coupling of the dimension 5 operator associated with the first Higgs in the 2HDM.
- RGEκ2 is the coupling of the dimension 5 operator associated with the second Higgs in the 2HDM.
- RGEλ1, RGEλ2, RGEλ3, RGEλ4 and RGEλ5 are the parameters λ_1, λ_2, λ_3, λ_4 and λ_5 in the Higgs potential (default: $\lambda_1 = \lambda_2 = 0.75$, $\lambda_3 = \lambda_4 = 0.2$, $\lambda_5 = 0.25$).
- RGEθ12, RGEθ13 and RGEθ23 are the angles θ_{12}, θ_{13} and θ_{23} of the MNS matrix in radians. (default: $\theta_{13} = 0$ and $\theta_{23} = \frac{\pi}{4}$). The default of θ_{12} depends on the model. It is chosen in such a way, that the parameters are compatible with the experimental data.
- RGEg RGEg is the coupling constants of SU(5)
- RGEg1, RGEg2 and RGEg3 are the coupling constants of $\mathrm{U(1)_Y}$, $\mathrm{SU(2)_L}$ and $\mathrm{SU(3)_C}$, respectively. GUT charge normalization is used for g_1.
- RGEm RGEm is the Higgs mass
- RGEqφ1 and RGEqφ2 are the unphysical phases φ_1 and φ_2 of the CKM matrix which correspond to the Majorana phases in the MNS matrix (default: 0).
- RGEqδ is the Dirac CP phase δ of the CKM matrix.
- RGEqδe, RGEq$\delta\mu$ and RGE$\delta\tau$ are the unphysical phases δ_e, δ_μ and δ_τ of the CKM matrix (default: 0).
- RGEqθ12, RGEqθ13 and RGEqθ23 are the angles of the CKM matrix.
- RGEyd, RGEys and RGEyb are the Yukawa coupling of the down-type quarks d, s and b.
- RGEye, RGEyμ and RGEyτ are the Yukawa couplings of the charged leptons e, μ and τ.
- RGEyu, RGEyc and RGEyt are the Yukawa couplings of the up-type quarks u, c and t.

Parameters accepted by `RGEGetSolution`:

- RGECoupling is used to get the coupling constants.
- RGEGWCondition returns the Gildener Weinberg condition.
- RGEGWConditions returns all Gildener Weinberg conditions.
- RGEM1Tilde returns the effective light-neutrino mass $\widetilde{m}_1 = \frac{\left(m_D m_D^\dagger\right)_{11}}{M_1} = \frac{(Y_\nu Y_\nu^\dagger)_{11} v^2}{2M_1}$ which is commonly used in thermal leptogenesis. $\widetilde{m}_1$ is given in eV.
- RGEMν is used to get the mass matrix of the left-handed neutrinos.
- RGEMνr is the mass matrix of the right-handed neutrinos.
- RGEMd is used to get the mass matrix of the down-type quarks.
- RGEMe is used to get the mass matrix of the charged leptons.
- RGEMixingParameters returns the mixing parameters in the leptonic sector as they are returned by MNSParameters: $\{\{\theta_{12}, \theta_{13}, \theta_{23}, \delta, \delta_e, \delta_\mu, \delta_\tau, \varphi_1, \varphi_2\}, \{y_1, y_2, y_3\}, \{y_e, y_\mu, y_\tau\}\}$

- RGEMu is used to get the mass matrix of the up-type quarks.
- RGEPoleMTop is used to get the pole mass of the top quark in the $\overline{\text{MS}}$ scheme. The pole mass term of the top quark is given by

$$m_t^{\text{Pole}} = m_t(m_t) \cdot (1 + \frac{4\alpha_s}{3\pi}) \tag{5.10}$$

  to 1-loop order.
- RGERawMνr is used to get the raw mass matrix of the right-handed neutrinos.
- RGERaw is used to get the raw values of all parameters. A raw parameter is the internal representation of the parameter
- RGERawYΔ is used to get the Yukawa coupling matrix of the coupling to the Higgs triplet.
- RGERawYν is used to get the raw Yukawa coupling matrix of the neutrinos.
- RGEAll returns all parameters of the model.
- RGEVEVratio returns the squared ratio of v_R over the EW symmetry breaking scale.
- RGEVEVratios returns the squared ratio of v_R over the EW symmetry breaking scale.
- RGEYν is used to get the Yukawa coupling matrix of the neutrinos.
- RGEYd is used to get the Yukawa coupling matrix of the down-type quarks.
- RGEYe is used to get the Yukawa coupling matrix of the charged leptons.
- RGEYu is used to get the Yukawa coupling matrix of the up-type quarks.
- RGEα is used to get the fine structure constants.
- RGEκ1 is the parameter of the dimension 5 operator associated with the first Higgs in the 2HDM.
- RGEκ2 is the parameter of the dimension 5 operator associated with the second Higgs in the 2HDM.
- RGEλ is used to get the Higgs couplings.

5.3.2 `REAP‘RGE2HDMON‘`

This package contains the Two Higgs Doublet Model (2HDM) with a $\mathbb{Z}_2$ symmetry without right-handed neutrinos.

It has the same parameters and options as `RGE2HDM`, with the following exceptions: The only missing options are RGEIntegratedOut, RGESearchTransition, RGEThresholdFactor, RGEPrecision and RGEMaxNumberIterations, which are used to control the process of integrating out. In addition RGEMνr and RGEYν are no parameters of `RGESetInitial` and RGEM1Tilde, RGERawMνr and RGERawYν are not accepted as parameters by `RGEGetSolution`. `RGESetInitial` has an additional option: RGESuggestion can be used to choose between different sets of default values, “GUT” (default) and “MZ”. They refer to typical parameter values at the GUT scale or at the Z mass, respectively.

5.3.3 `REAP‘RGE2HDMDirac‘`

This package contains the 2HDM with Dirac neutrinos to 1 loop order.

It has the same parameters and options as `RGE2HDM`, with the following exceptions: The only missing options are RGEIntegratedOut, RGESearchTransition, RGEThresholdFactor, RGEPrecision and RGEMaxNumberIterations, which are used to control the process of integrating out. In addition RGEMνr, RGEκ1 and RGEκ2 are no parameter of `RGESetInitial` and RGEMixingParameters, RGEM1Tilde, RGERawMνr, RGERawYν, RGEκ1 and RGEκ2 are not accepted as parameters by `RGEGetSolution`. `RGESetInitial` has an additional option: RGESuggestion can be used to choose between different sets of default values, “GUT” (default) and “MZ”. They refer to typical parameter values at the GUT scale or at the Z mass, respectively.

6 Conventions

6.1 Definition of the parameters

6.1.1 GUT Charge Normalization

We use the GUT charge normalization in all models which is related to the charge normalization in the SM by

$$q_Y^{\mathrm{GUT}} = \sqrt{\frac{3}{5}} q_Y^{\mathrm{SM}} . \tag{6.1}$$

Therefore the coupling constant satisfies

$$\left(g_1^{\mathrm{SM}}\right)^2 = \frac{3}{5}\left(g_1^{\mathrm{GUT}}\right)^2 . \tag{6.2}$$

6.1.2 Convention for Yukawa Matrices

We use the RL convention for Yukawa matrices, i.e.

$$\mathscr{L}_{\mathrm{Yukawa}} = -Y_{ij}\overline{\psi_{\mathrm{R}}^i}\psi_{\mathrm{L}}^j \cdot \phi + \mathrm{h.c.} . \tag{6.3}$$

6.1.3 Vacuum Expectation Value of the SM Higgs

The vacuum expectation value of the SM Higgs is $v = 246\,\mathrm{GeV}$. It is assumed to be constant.

6.1.4 Standard Parameterization of the Lepton Mixing Matrix

A unitary matrix can be described by 3 angles and 6 phases. Thus it can be written in the following way:

$$U = \mathrm{diag}(e^{\mathrm{i}\delta_e}, e^{\mathrm{i}\delta_\mu}, e^{\mathrm{i}\delta_\tau}) \cdot V(\theta_{12},\theta_{13},\theta_{23}) \cdot \mathrm{diag}(e^{-\mathrm{i}\varphi_1/2}, e^{-\mathrm{i}\varphi_2/2}, 1) \tag{6.4}$$

V is a special unitary matrix and is parameterized in standard parameterization like the CKM matrix in the quark sector with 3 angles (θ_{12}, θ_{13}, θ_{23}) and 1 CP phase (δ).

$$V(\theta_{12},\theta_{13},\theta_{23}) = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}\,e^{-\mathrm{i}\delta} \\ -c_{23}s_{12} - s_{23}s_{13}c_{12}\,e^{\mathrm{i}\delta} & c_{23}c_{12} - s_{23}s_{13}s_{12}\,e^{\mathrm{i}\delta} & s_{23}c_{13} \\ s_{23}s_{12} - c_{23}s_{13}c_{12}\,e^{\mathrm{i}\delta} & -s_{23}c_{12} - c_{23}s_{13}s_{12}\,e^{\mathrm{i}\delta} & c_{23}c_{13} \end{pmatrix} \tag{6.5}$$

where s_{ij} and c_{ij} are defined as$s_{ij} = \sin\theta_{ij}$ and $c_{ij} = \cos\theta_{ij}$, respectively. In addition there are phase matrices multiplied from both sides. The matrix on the left-hand side is characterized by the unphysical phases δ_e, δ_μ and δ_τ which can be rotated away by a change of the phases in the charged left-handed leptons in the extended (MS)SM.

$$|\ell_L\rangle \to \mathrm{diag}(e^{-\mathrm{i}\delta_e}, e^{-\mathrm{i}\delta_\mu}, e^{-\mathrm{i}\delta_\tau})\,|\ell_L\rangle \tag{6.6}$$

The matrix on the right-hand side is described by the Majorana phases φ_1 and φ_2. These can not be rotated away by a redefinition of fields, because the effective neutrino mass term is a Majorana mass term which is diagonalized by an unitary transformation and not by a biunitary transformation like the Yukawa matrix of the charged leptons.

6.2 Naming conventions

6.2.1 Variable names

The first letter of each word and abbreviations like ("RGE") in a variable name are capitalized. The remaining letters are uncapitalized. All local variables have "l" as prefix and all parameters in functions have "p" as prefix.

6.2.2 Function names

The first letter of each word and abbreviations like ("RGE") in a function name are capitalized. The remaining letters are uncapitalized. All public functions in `REAP.m` begin with "RGE".

6.2.3 Exceptions

The first letter of each word and abbreviations like ("RGE") in the name of a exception are capitalized. The remaining letters are uncapitalized.
The defined exceptions are:

- RGEModelAlreadyRegistered will be thrown if the model already is registered.
- RGELessThanZero will be thrown if a parameter is less than zero, thus out of range.
- RGEScaleTooBig will be thrown if the scale parameter is too big, thus out of range.
- RGENotImplementedYet will be thrown if the transition function isn't implemented yet.
- RGEOutOfRange will be thrown if the parameter is out of range.
- RGEModelDoesNotExist will be thrown if the model name given does not exist in the list Model. Thus the model hasn't been registered yet.
- RGEWrongModel will be thrown if the model name does not corresponding to the model valid at the given scale.
- RGEνMassAboveCutoff will be thrown if an eigenvalue of Mν is above the cutoff.
- RGENotAValidMassHierarchy will be thrown if the type of the given mass hierarchy does not exist.

7 How to define a new model

A model has to provide several functions. In the following a simple example of a toy model with one running coupling constant is shown. It is contained in the file `RGEToyModel.m`.

(1) First of all it must have a function returning the parameters with no arguments.

```
Parameters={\[Lambda]};

ClearAll[GetParameters];
GetParameters[]:= Module[{},
   Return[Parameters];
];
```

(2) Furthermore there has to be a function to solve the RGE.

```
ClearAll[RGE];
RGE:={  D[\[Lambda][t],t]==Beta\[Lambda][\[Lambda][t]] };

ClearAll[Beta\[Lambda]];
Beta\[Lambda][\[Lambda]_] :=-7 * 1/(16*Pi^2) * \[Lambda]^3;

ClearAll[SolveModel];
SolveModel[{pUp_,pUpModel_,pUpOptions_},{pDown_,pDownModel_,pDownOptions_},
           pDirection_,pBoundary_,pInitial_,pOpts___]
           :=Module[{lSolution,lNDSolveOpts,lNewScale,lInitial},
        lNDSolveOpt;
        Options[lNDSolveOpts]=Options[NDSolve];
        SetOptions[lNDSolveOpts,FilterOptions[NDSolve,Options[RGEOptions]]];
        SetOptions[lNDSolveOpts,FilterOptions[NDSolve,pOpts]];
        lInitial=SetInitial[pBoundary,pInitial];
        lSolution=NDSolve[RGE ~Join~ lInitial, Parameters,{t,pDown,pUp},
                Sequence[Options[lNDSolveOpts]]];
        If[lDirection>0,lNewScale=pUp,lNewScale=pDown];
        Return[{lSolution,lNewScale,0}];
];
```

The arguments of `SolveModel` are

- pUp is the upper bound. It is the logarithm of the renormalization scale $\log \mu$.
- pUpModel is the modelname of the model which is valid above pUp.
- pUpOptions are the options of the model which is valid above pUp.
- pDown, pDownModel and pDownOptions are the corresponding options for the lower bound.
- pDirection specifies whether the RGEs are solved upwards or downwards.
- pBoundary is the scale where the initial values are given. It is the logarithm of the renormalization scale scale $\log \mu$.
- pInitial is the list of initial values which given as replacement rules.
- pOpts are options for NDSolve,. . . .

(3) The transition functions provide the possibility to implement a model which has several transitions to other EFT's (e.g. integrating out degrees of freedom like heavy right handed neutrinos):

```
ClearAll[Transition];
Transition[pScale_?NumericQ,pDirection_?NumericQ,pSolution_,pToOpts_,pFromOpts_]
          :=Module[{},
       Return[({RGE\[Lambda]->\[Lambda][pScale]}/.pSolution)[[1]]];
];
```

(4) The model has to provide initial values. The only argument of the function are the specified initial values of the user. The initial values are returned as replacement rules.

```
ClearAll[GetInitial];
GetInitial[pOpts___:{}]:=Module[{lInitial,l\[Lambda]},
        lInitial={RGE\[Lambda]->0.1};
```

```
        l\[Lambda]=(RGE\[Lambda]/.pOpts)/.lInitial;
        Return[{RGE\[Lambda]->l\[Lambda]}];
];
```

(5) The model has to provide functions to set and return options. The function to set the options takes the options as parameters and the function returning the options doesn't have any parameters.

```
ClearAll[ModelSetOptions];
ModelSetOptions[pOpts_]:= Module[{},
    SetOptions[RGEOptions,FilterOptions[RGEOptions,pOpts]];
];

ClearAll[ModelGetOptions];
ModelGetOptions[]:= Module[{},
   Return[Options[RGEOptions]];
];
```

(6) Finally the model has to provide functions to return the solution which take as argument the logarithmic energy scale $\log\mu$ and the solution for this energy range. `pOpts` might contain options which are relevant for the model.

```
ClearAll[GetSolution];
GetSolution[pScale_,pSolution_,pOpts___]:=Module[{},
        Return[({\[Lambda][pScale]}/.pSolution)[[1]]];
];
```

At last the model has to be registered.
The function `RGERegisterModel[name, function returning parameters, solution, list of transition functions, provide initial values, set options, get options]` of the package `REAP` is used to register a new model (see 4.2.12).

```
RGERegisterModel["Toy","REAP`Toy`",
        `Private`GetParameters,
        `Private`SolveModel,
        {RGEAll->`Private`GetSolution},
        {{"Toy",`Private`Transition}},
        `Private`GetInitial,
        `Private`ModelSetOptions,
        `Private`ModelGetOptions
];
```

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