VINCIA Authors' Compendium

P. Skands, W. T. Giele, L. Hartgring, D. A. Kosower, E. Laenen,
A. Larkoski, J. J. Lopez-Villarejo, M. Ritzmann

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A Evolution Equations

A.1 Final–Final Evolution Variables

The evolution variables considered in VINCIA for final–final antennae are the following [1, 2]:

$$Q_{\perp}^2 = N_{\perp}\frac{s_{ij}s_{jk}}{m_{IK}^2} = N_{\perp}\, y_{ij}\, y_{jk}\, m_{IK}^2 = N_{\perp}\, p_{\perp A}^2 \,, \tag{1}$$

$$m_D^2 = N_D \min(s_{ij}, s_{jk}) = N_D \min(y_{ij}, y_{jk})\, m_{IK}^2 \,, \tag{2}$$

$$E^{*2} = \frac{(s_{ij}+s_{jk})^2}{m_{IK}^2} = (y_{ij}+y_{jk})^2\, m_{IK}^2 \,, \tag{3}$$

$$m_{g^*}^2 = m_{jk}^2 \,, \tag{4}$$

with the arbitrary normalization factors $N_{\perp} \in [1,4]$ and $N_D \in [1,2]$, the invariant mass

$$m_{IK}^2 \;=\; (p_I + p_K)^2 \;=\; (p_i + p_j + p_k)^2 \,, \tag{5}$$

and the symbol s_{ij} defined as the dot product

$$s_{ij} \;\equiv\; 2p_i\cdot p_j \;=\; (p_i+p_j)^2 - m_i^2 - m_j^2 \;\stackrel{m=0}{=}\; m_{ij}^2 \,. \tag{6}$$

The maximum values that these evolution variables attain on the physical final-final antenna phase-space are:

$$Q_{\perp\max}^2 = \frac{N_{\perp}}{4} m_{IK}^2 \,, \tag{7}$$

$$m_{D\max}^2 = \frac{N_D}{2} m_{IK}^2 \,, \tag{8}$$

$$E_{\max}^{*2} = m_{IK}^2 \,, \tag{9}$$

$$m_{g^*\max}^2 = m_{IK}^2 \,. \tag{10}$$

Note on dimensionality: the dimensionless form of the evolution variable is $y = Q^2/m_{IK}^2$, with Q denoting the choice of evolution variable among the above possibilities.

Note for m_D: the expressions below correspond to the branch with $y_{ij} < y_{jk}$ and hence will only generate branchings over half of phase space. For trial antenna functions symmetric in the invariants (specifically the soft-eikonal and hard-finite ones, see sec. A.4), the trial generation is done by multiplying the kernel by a factor 2 and randomly keeping or swapping the generated invariants. For the I- and K-collinear sector terms, we use that they are mutually related by $i \leftrightarrow k$, and hence an I-collinear term over all of phase space can be composed from an I-collinear one on the branch $y_{ij} < y_{jk}$ combined with a K-collinear one with swapped invariants on the complementary branch.

A.2 Zeta Definitions

The following choices of ζ are used:

$$\zeta_1 = \frac{y_{ij}}{y_{ij} + y_{jk}} \tag{11}$$

$$\zeta_2 = y_{ij} \tag{12}$$

$$\zeta_3 = y_{jk} \; . \tag{13}$$

The final–final phase-space limits are, for $Q_\perp$:

$$\zeta_{1\pm}(Q_\perp^2) = \frac{1}{2}\left(1 \pm \sqrt{1 - \frac{4}{N_\perp}\frac{Q_\perp^2}{m_{IK}^2}}\right) = \frac{1}{2}\left(1 \pm \sqrt{1 - 4\, y_{ij}\, y_{jk}}\right) \; , \tag{14}$$

$$\zeta_{2\pm}(Q_\perp^2) = \zeta_{1\pm}(Q_\perp^2) \; , \tag{15}$$

$$\zeta_{3\pm}(Q_\perp^2) = \zeta_{1\pm}(Q_\perp^2) \; , \tag{16}$$

for m_D:

$$\zeta_{1-}(m_D^2) = \frac{1}{2} \; , \tag{17}$$

$$\zeta_{1+}(m_D^2) = 1 - \frac{m_D^2}{N_D m_{IK}^2} = 1 - y_{ij} \; , \tag{18}$$

$$\zeta_{2-}(m_D^2) = \text{N/A} \; , \tag{19}$$

$$\zeta_{2+}(m_D^2) = \text{N/A} \; , \tag{20}$$

$$\zeta_{3-}(m_D^2) = \frac{m_D^2}{N_D m_{IK}^2} = y_{ij} \; , \tag{21}$$

$$\zeta_{3+}(m_D^2) = 1 - \frac{m_D^2}{N_D m_{IK}^2} = 1 - y_{ij} \; , \tag{22}$$

for E^*:

$$\zeta_\pm(E^{*2}) = \text{Special: see below} \tag{23}$$

for m_{g^*}:

$$\zeta_{2-}(m^2_{q\bar{q}}) = 0 \; , \tag{24}$$

$$\zeta_{2+}(m^2_{q\bar{q}}) = 1 - \frac{m^2_{g^*}}{m^2_{IK}} = 1 - y_{jk} \; . \tag{25}$$

Note that the phase-space limits for E^* coincide with the collinear limits. Integrations over any finite interval of E^* over the full allowed ζ range would therefore yield infinities. When using E^*-ordering, it is necessary to impose a hadronization cutoff in a complementary variable, such as $Q_\perp$ or m_D. This cutoff then defines the ζ boundaries for the integrations.

A.3 Jacobians

The Jacobians for the transformation from the original LIPS variables, (s_{ij}, s_{jk}), to the shower variables, (Q^2, ζ), are written as a product of a normalization-and-Q-dependent piece and a ζ-dependent factor,

$$|J| = J_Q \times J_\zeta \; . \tag{26}$$

They are, for $Q^2_\perp$:

$$|J(Q^2_\perp, \zeta_1)| = \frac{1}{2N_\perp} \times \frac{m^2_{IK}}{\zeta_1(1-\zeta_1)} \; , \tag{27}$$

$$|J(Q^2_\perp, \zeta_2)| = \frac{1}{N_\perp} \times \frac{m^2_{IK}}{\zeta_2} \; , \tag{28}$$

$$|J(Q^2_\perp, \zeta_3)| = \frac{1}{N_\perp} \times \frac{m^2_{IK}}{\zeta_3} \; , \tag{29}$$

for m^2_D:

$$|J(m^2_D, \zeta_1)| = \text{Not Used} \; , \tag{30}$$

$$|J(m^2_D, \zeta_2)| = \text{N/A} \; , \tag{31}$$

$$|J(m^2_D, \zeta_3)| = \frac{1}{N_D} \times m^2_{IK} \; , \tag{32}$$

for E^{*2}:

$$|J(E^{*2}, \zeta_1)| = \frac{1}{2} \times m^2_{IK} \; , \tag{33}$$

$$|J(E^{*2}, \zeta_2)| = \frac{m_{IK}}{2\sqrt{E^{*2}}} \times m^2_{IK} \; , \tag{34}$$

$$|J(E^{*2}, \zeta_3)| = \frac{m_{IK}}{2\sqrt{E^{*2}}} \times m^2_{IK} \; , \tag{35}$$

for m_{g^*}:

$$|J(m_{q\bar{q}}^2, \zeta_2)| = 1 \times m_{IK}^2 \ . \tag{36}$$

A.4 Trial Functions

The following trial functions are available:

$$\text{Eikonal (soft)} : \hat{a}_E = \frac{1}{m_{IK}^2} \frac{2}{y_{ij} y_{jk}} \tag{37}$$

$$\text{Constant (hard)} : \hat{a}_F = \frac{1}{m_{IK}^2} \tag{38}$$

$$\text{I Collinear (sector)} : \hat{a}_I = \frac{1}{m_{IK}^2} \frac{2}{y_{ij}(1 - y_{jk})} \tag{39}$$

$$\text{K Collinear (sector)} : \hat{a}_K = \frac{1}{m_{IK}^2} \frac{2}{y_{jk}(1 - y_{ij})} \tag{40}$$

$$\text{K Splitting } (g \to q\bar{q}) : \hat{a}_S = \frac{1}{m_{IK}^2} \frac{1}{y_{jk}} \ , \tag{41}$$

A.5 Zeta Integrals

For a given trial antenna function, $\hat{a}$, the definition of the ζ integral is:

$$I_\zeta = \int_{\zeta_a}^{\zeta_b} d\zeta \ J_\zeta \ \hat{a} \tag{42}$$

where $|J_\zeta|$ signifies the part of the Jacobian that only has ζ dependence (see above), and $\zeta_b > \zeta_a$ represents an arbitrary ζ interval. This interval will in general be larger than the physically allowed one (trials generated outside the physical phase space will be rejected by a veto). We shall nevertheless still assume that all ζ values are at least inside the range $\zeta \in [0, 1]$.

The integration kernels are, for $Q_\perp$:

$$J_\zeta \ \hat{a}_{E,F}(Q_\perp^2, \zeta_1) = \frac{1}{(1 - \zeta_1)\zeta_1} \ , \tag{43}$$

$$J_\zeta \ \hat{a}_I(Q_\perp^2, \zeta_3) = \frac{1}{(1 - \zeta_3)} \ , \tag{44}$$

$$J_\zeta \ \hat{a}_K(Q_\perp^2, \zeta_2) = \frac{1}{(1 - \zeta_2)} \ , \tag{45}$$

for m_D:

$$J_\zeta\, \hat{a}_E(m_D^2, \zeta_3) = \frac{1}{\zeta_3}\ , \tag{46}$$

$$J_\zeta\, \hat{a}_F(m_D^2, \zeta_3) = 1\ , \tag{47}$$

$$J_\zeta\, \hat{a}_I(m_D^2, \zeta_3) = \frac{1}{(1-\zeta_3)}\ , \tag{48}$$

for E^*:

$$J_\zeta\, \hat{a}_E(E^{*2}, \zeta_1) = \frac{1}{\zeta_1^2}\ , \tag{49}$$

$$J_\zeta\, \hat{a}_F(E^{*2}, \zeta_1) = \frac{1}{2}\ , \tag{50}$$

$$\tag{51}$$

for m_{g^*}:

$$J_\zeta \hat{a}_S(m_{g^*}^2, \zeta_2) = 1 \tag{52}$$

with integrals over the range $\zeta_a < \zeta_b$, for $Q_\perp$:

$$I_{\zeta E,F}(Q_\perp^2, \zeta_1) = \ln\left(\frac{\zeta_b(1-\zeta_a)}{\zeta_a(1-\zeta_b)}\right)\ , \tag{53}$$

$$I_{\zeta I}(Q_\perp^2, \zeta_3) = \ln\left(\frac{1-\zeta_a}{1-\zeta_b}\right)\ , \tag{54}$$

$$I_{\zeta K}(Q_\perp^2, \zeta_2) = \ln\left(\frac{1-\zeta_a}{1-\zeta_b}\right)\ , \tag{55}$$

for m_D:

$$I_{\zeta E}(m_D^2, \zeta_3) = \ln\left(\frac{\zeta_b}{\zeta_a}\right)\ , \tag{56}$$

$$I_{\zeta F}(m_D^2, \zeta_3) = \zeta_b - \zeta_a\ , \tag{57}$$

$$I_{\zeta I}(m_D^2, \zeta_3) = \ln\left(\frac{1-\zeta_a}{1-\zeta_b}\right)\ , \tag{58}$$

for E^*:

$$I_{\zeta E}(E^{*2}, \zeta_1) = \frac{1}{z_a} - \frac{1}{z_b}\ , \tag{59}$$

$$I_{\zeta F}(E^{*2}, \zeta_1) = \zeta_b - \zeta_a\ , \tag{60}$$

for m_{g^*}:

$$I_{\zeta S}(m_{g^*}^2, \zeta_2) = \zeta_b - \zeta_a \ . \tag{61}$$

A.6 Evolution Integrals

The evolution integral, for a particular choice of Q and ζ, is defined as follows

$$\hat{\mathcal{A}}(Q_1^2, Q_2^2) = \int_{Q_2^2}^{Q_1^2} dQ^2 \, \frac{\mathcal{C} \, g_s^2}{16\pi^2 m_{IK}^2} \, J_Q(Q,\zeta) \, I_\zeta(Q,\zeta) \ , \tag{62}$$

with $\mathcal{C}$ the (trial) color factor (typically C_A for gluon emission and 1 for gluon splitting) and J_Q the non-ζ dependent part of the Jacobian, see eqs. (27) – (36).

Note: for massive partons, the phase-space factor should actually be larger: m_{IK} in the denominator should be replaced by the Källén function, see [3]. This is taken care of during trial generation by applying an overall prefactor representing the phase-space volume and using the same integrals as shown here below.

We label the integrand in the above equation by

$$d\hat{\mathcal{A}} = \frac{\mathcal{C} \, g_s^2}{16\pi^2 s} \, J_Q(Q,\zeta) \, I_\zeta(Q,\zeta) \ , \tag{63}$$

which takes the following specific forms, for $Q_\perp$:

$$d\hat{\mathcal{A}}_E(Q_\perp^2) = \frac{\hat{\alpha}_s}{4\pi} \, \mathcal{C} \, I_{\zeta E}(Q_\perp^2, \zeta_1) \, \frac{1}{Q_\perp^2} \ , \tag{64}$$

$$d\hat{\mathcal{A}}_F(Q_\perp^2) = \frac{1}{2N_\perp} \, \frac{\hat{\alpha}_s}{4\pi} \, \mathcal{C} \, I_{\zeta F}(Q_\perp^2, \zeta_1) \, \frac{1}{m_{IK}^2} \ , \tag{65}$$

$$d\hat{\mathcal{A}}_I(Q_\perp^2) = 2 \, \frac{\hat{\alpha}_s}{4\pi} \, \mathcal{C} \, I_{\zeta I}(Q_\perp^2, \zeta_3) \, \frac{1}{Q_\perp^2} \ , \tag{66}$$

$$d\hat{\mathcal{A}}_K(Q_\perp^2) = 2 \, \frac{\hat{\alpha}_s}{4\pi} \, \mathcal{C} \, I_{\zeta K}(Q_\perp^2, \zeta_2) \, \frac{1}{Q_\perp^2} \ , \tag{67}$$

for m_D:

$$d\hat{\mathcal{A}}_{E,I}(m_D^2) = \frac{\hat{\alpha}_s}{4\pi} \, \mathcal{C} \, 2 I_{\zeta E,I}(m_D^2, \zeta_3) \, \frac{1}{m_D^2} \ , \tag{68}$$

$$d\hat{\mathcal{A}}_F(m_D^2) = \frac{1}{N_D} \, \frac{\hat{\alpha}_s}{4\pi} \, \mathcal{C} \, I_{\zeta F}(m_D^2, \zeta_3) \, \frac{1}{m_{IK}^2} \ , \tag{69}$$

for E^*:

$$d\hat{\mathcal{A}}_E(E^{*2}) = \frac{\hat{\alpha}_s}{4\pi}\,\mathcal{C}\,I_{\zeta E}(E^{*2},\zeta_1)\,\frac{1}{\sqrt{E^{*2}m_{IK}^2}}\ , \tag{70}$$

$$d\hat{\mathcal{A}}_F(E^{*2}) = \frac{\hat{\alpha}_s}{4\pi}\,\mathcal{C}\,\frac{1}{2}I_{\zeta F}(E^{*2},\zeta_1)\,\frac{1}{m_{IK}^2}\ , \tag{71}$$

for m_{g^*}:

$$d\hat{\mathcal{A}}_S(m_{g^*}^2) = \frac{\hat{\alpha}_s}{4\pi}\,\mathcal{C}\,I_{\zeta S}(m_{g^*}^2,\zeta_2)\,\frac{1}{m_{g^*}^2}\ . \tag{72}$$

For a constant trial $\hat{\alpha}_s$, the evolution integrals are, for $Q_\perp$:

$$\hat{\mathcal{A}}_E^0(Q_{\perp 1}^2,Q_{\perp 2}^2) = \frac{\hat{\alpha}_s}{4\pi}\,\mathcal{C}\,I_{\zeta E}\ \ln\left(\frac{Q_{\perp 1}^2}{Q_{\perp 2}^2}\right)\ , \tag{73}$$

$$\hat{\mathcal{A}}_F^0(Q_{\perp 1}^2,Q_{\perp 2}^2) = \frac{1}{2N_\perp}\,\frac{\hat{\alpha}_s}{4\pi}\,\mathcal{C}\,I_{\zeta F}\,\frac{(Q_{\perp 1}^2-Q_{\perp 2}^2)}{m_{IK}^2}\ , \tag{74}$$

$$\hat{\mathcal{A}}_{I,K}^0(Q_{\perp 1}^2,Q_{\perp 2}^2) = 2\,\frac{\hat{\alpha}_s}{4\pi}\,\mathcal{C}\,I_{\zeta I,K}\ \ln\left(\frac{Q_{\perp 1}^2}{Q_{\perp 2}^2}\right)\ , \tag{75}$$

for m_D:

$$\hat{\mathcal{A}}_{E,I}^0(m_{D1}^2,m_{D2}^2) = \frac{\hat{\alpha}_s}{4\pi}\,\mathcal{C}\,2I_{\zeta E,I}\ \ln\left(\frac{m_{D1}^2}{m_{D2}^2}\right)\ , \tag{76}$$

$$\hat{\mathcal{A}}_F^0(m_{D1}^2,m_{D2}^2) = \frac{1}{N_D}\,\frac{\hat{\alpha}_s}{4\pi}\,\mathcal{C}\,I_{\zeta F}\,\frac{(m_{D1}^2-m_{D2}^2)}{m_{IK}^2}\ , \tag{77}$$

for E^*:

$$\hat{\mathcal{A}}_E^0(E_1^{*2},E_2^{*2}) = \frac{\hat{\alpha}_s}{4\pi}\,\mathcal{C}\,2I_{\zeta E}\,\frac{\left(\sqrt{E_1^{*2}}-\sqrt{E_2^{*2}}\right)}{m_{IK}}\ , \tag{78}$$

$$\hat{\mathcal{A}}_F^0(E_1^{*2},E_2^{*2}) = \frac{\hat{\alpha}_s}{4\pi}\,\mathcal{C}\,\frac{1}{2}I_{\zeta F}\,\frac{(E_1^{*2}-E_2^{*2})}{m_{IK}^2} \tag{79}$$

for m_{g*}:

$$\hat{\mathcal{A}}_S^0(m_{g1}^2,m_{g2}^2) = \frac{\hat{\alpha}_s}{4\pi}\,\mathcal{C}\,I_{\zeta S}\ \ln\left(\frac{m_{g1}^2}{m_{g2}^2}\right)\ . \tag{80}$$

For a first-order running trial $\hat{\alpha}_s(Q^2)$,

$$\hat{\alpha}_s(Q^2_\perp) = \frac{1}{b_0 \ln\left(\frac{k_R^2 p_{\perp A}^2}{\Lambda^2}\right)} = \frac{1}{b_0 \ln\left(\frac{k_R^2 Q_\perp^2}{N_\perp \Lambda^2}\right)} , \tag{81}$$

$$\hat{\alpha}_s(m^2_D) = \frac{1}{b_0 \ln\left(\frac{k_R^2 m_{\min}^2}{\Lambda^2}\right)} = \frac{1}{b_0 \ln\left(\frac{k_R^2 m_D^2}{N_D \Lambda^2}\right)} , \tag{82}$$

$$\hat{\alpha}_s(m^2_{g^*}) = \frac{1}{b_0 \ln\left(\frac{k_R^2 m_{g^*}^2}{\Lambda^2}\right)} , \tag{83}$$

with k_R an arbitrary scaling factor that includes the compound effect of any renormalization-scale prefactor choices and the optional translation between the MSbar and CMW schemes for Λ, the evolution integrals are, for $Q_\perp$:

$$\hat{\mathcal{A}}^1_E(Q^2_{\perp 1}, Q^2_{\perp 2}) = \frac{\mathcal{C} I_{\zeta E}}{4\pi b_0} \ln\left(\frac{\ln\left(\frac{k_R^2 Q_{\perp 1}^2}{N_\perp \Lambda^2}\right)}{\ln\left(\frac{k_R^2 Q_{\perp 2}^2}{N_\perp \Lambda^2}\right)}\right) , \tag{84}$$

$$\hat{\mathcal{A}}^1_F(Q^2_{\perp 1}, Q^2_{\perp 2}) = \text{Not Used (generates LogIntegrals)} , \tag{85}$$

$$\hat{\mathcal{A}}^1_{I,K}(Q^2_{\perp 1}, Q^2_{\perp 2}) = 2\,\frac{\mathcal{C} I_{\zeta I,K}}{4\pi b_0} \ln\left(\frac{\ln\left(\frac{k_R^2 Q_{\perp 1}^2}{N_\perp \Lambda^2}\right)}{\ln\left(\frac{k_R^2 Q_{\perp 2}^2}{N_\perp \Lambda^2}\right)}\right) , \tag{86}$$

for m_D:

$$\hat{\mathcal{A}}^1_{E,I}(m^2_{D1}, m^2_{D2}) = 2\frac{\mathcal{C} I_{\zeta E,I}}{4\pi b_0} \ln\left(\frac{\ln\left(\frac{k_R^2 m_{D1}^2}{N_D \Lambda^2}\right)}{\ln\left(\frac{k_R^2 m_{D2}^2}{N_D \Lambda^2}\right)}\right) , \tag{87}$$

$$\hat{\mathcal{A}}^1_F(m^2_{D1}, m^2_{D2}) = \text{Not Used (generates LogIntegrals)} , \tag{88}$$

for m_g^*:

$$\hat{\mathcal{A}}^1_S(m^2_{g1}, m^2_{g2}) = \frac{\mathcal{C} I_{\zeta S}}{4\pi b_0} \ln\left(\frac{\ln\left(\frac{k_R^2 m_{g1}^2}{\Lambda^2}\right)}{\ln\left(\frac{k_R^2 m_{g2}^2}{\Lambda^2}\right)}\right) . \tag{89}$$

A.7 Generation of Trial Evolution Scale

The trial Sudakov factor is defined as:

$$\hat{\Delta}(Q_1^2, Q_2^2) = \exp\left[-\hat{\mathcal{A}}(Q_1^2, Q_2^2)\right] , \tag{90}$$

and the next trial scale is found by solving the equation:

$$\mathcal{R} = \hat{\Delta}(Q^2, Q^2_{\rm new}) \ , \tag{91}$$

for $Q_{\rm new}$, with $\mathcal{R}$ a random number distributed uniformly in the interval $\mathcal{R} \in [0,1]$, and Q the current "restart scale". For strongly ordered showers, the restart scale after an accepted trial branching is the evolution scale evaluated on the current parton configuration. For smoothly ordered showers, this restart scale is only used for antennae that are not color-adjacent to the branching that occurred; for the newly created antennae, and (optionally) for any color-adjacent ones, the restart scale is the respective antenna invariant masses[1].

For both strongly and smoothly ordered showers, the restart scale after a failed (vetoed) trial branching is the scale of the failed branching.

Note: to optimize event generation, trial scales can be saved and reused for any antennae whose flavors, spins, and invariant masses are preserved by the preceding branching step.

For constant trial $\hat{\alpha}_s$, the solutions for the next trial scale are, for $Q_\perp$:

$$Q^2_{\perp E{\rm new}} = Q^2_\perp \mathcal{R}^{\frac{4\pi}{\hat{\alpha}_s}\frac{1}{\mathcal{C}I_{\zeta E}}} \ , \tag{92}$$

$$Q^2_{\perp F{\rm new}} = Q^2_\perp - m^2_{IK}\, 2N_\perp \frac{4\pi}{\hat{\alpha}_s}\frac{1}{\mathcal{C}I_{\zeta F}} \ln(1/\mathcal{R}) \ , \tag{93}$$

$$Q^2_{\perp I,K{\rm new}} = Q^2_\perp \mathcal{R}^{\frac{1}{2}\frac{4\pi}{\hat{\alpha}_s}\frac{1}{\mathcal{C}I_{\zeta I,K}}} \ , \tag{94}$$

for m_D:

$$m^2_{DE,I{\rm new}} = m^2_D \mathcal{R}^{\frac{4\pi}{\hat{\alpha}_s}\frac{1}{2\mathcal{C}I_{\zeta E,I}}} \ , \tag{95}$$

$$m^2_{DF{\rm new}} = m^2_D - m^2_{IK}\, N_D \frac{4\pi}{\hat{\alpha}_s}\frac{1}{\mathcal{C}I_{\zeta F}} \ln(1/\mathcal{R}) \ , \tag{96}$$

for E^*:

$$E^{*2}_{E{\rm new}} = \left(\sqrt{E^{*2}} - m_{IK}\frac{4\pi}{\hat{\alpha}_s}\frac{1}{\mathcal{C}I_{\zeta E}} \ln(1/\mathcal{R})\right)^2 \ , \tag{97}$$

$$E^{*2}_{F{\rm new}} = E^{*2} - m^2_{IK}\frac{4\pi}{\hat{\alpha}_s}\frac{1}{2\mathcal{C}I_{\zeta F}} \ln(1/\mathcal{R}) \ , \tag{98}$$

for m_D:

$$m^2_{g^*S{\rm new}} = m^2_{g^*}\mathcal{R}^{\frac{4\pi}{\hat{\alpha}_s}\frac{1}{\mathcal{C}I_{\zeta S}}} \ . \tag{99}$$

[1] This allows hard $2 \to n$ branchings to be generated inside the newly created antennae (and optionally within the color-adjacent ones) without disturbing the evolution of the rest of the event.

For a one-loop running trial $\hat{\alpha}_s(\mu_R^2)$, with $\mu_R^2 \propto Q^2$, the solutions for the next trial scale are, for $Q_\perp$:

$$\ln\left(\frac{k_R^2 Q_{\perp E\mathrm{new}}^2}{N_\perp \Lambda^2}\right) = \mathcal{R}^{\frac{4\pi b_0}{C I_{\zeta E}}} \ln\left(\frac{k_R^2 Q_\perp^2}{N_\perp \Lambda^2}\right) , \tag{100}$$

$$\ln\left(\frac{k_R^2 Q_{\perp I,K\mathrm{new}}^2}{N_\perp \Lambda^2}\right) = \mathcal{R}^{\frac{1}{2}\frac{4\pi b_0}{C I_{\zeta E}}} \ln\left(\frac{k_R^2 Q_\perp^2}{N_\perp \Lambda^2}\right) , \tag{101}$$

for m_D:

$$\ln\left(\frac{k_R^2 m_{DE,I\mathrm{new}}^2}{N_D \Lambda^2}\right) = \mathcal{R}^{\frac{4\pi b_0}{2C I_{\zeta E,I}}} \ln\left(\frac{k_R^2 m_D^2}{N_D \Lambda^2}\right) , \tag{102}$$

for m_{g^*}:

$$\ln\left(\frac{k_R^2 m_{g^* S\mathrm{new}}^2}{\Lambda^2}\right) = \mathcal{R}^{\frac{4\pi b_0}{C I_{\zeta S}}} \ln\left(\frac{k_R^2 m_{g^*}^2}{\Lambda^2}\right) . \tag{103}$$

A.8 Generation of Trial Zeta

The trial value for ζ is found by inverting the equation

$$\mathcal{R}_\zeta = \frac{I_\zeta(\zeta_{\mathrm{min}}, \zeta)}{I_\zeta(\zeta_{\mathrm{min}}, \zeta_{\mathrm{max}})} , \tag{104}$$

where the boundary values $(\zeta_{\mathrm{min}}, \zeta_{\mathrm{max}})$ must be the same as those that were used to evaluate the I_ζ integrals during the generation of the trial scale above, i.e., they must correspond to the phase-space overestimate used for the trial generation. The forms of I_ζ are given for each evolution variable separately in eqs. (53)–(60).

For $Q_\perp$, the solutions to eq. (104) are:

$$\zeta_{1E,F}(\mathcal{R}) = \left[1 + \frac{1-\zeta_{\mathrm{min}}}{\zeta_{\mathrm{min}}}\left(\frac{\zeta_{\mathrm{min}}(1-\zeta_{\mathrm{max}})}{\zeta_{\mathrm{max}}(1-\zeta_{\mathrm{min}})}\right)^{\mathcal{R}}\right]^{-1} , \tag{105}$$

$$\zeta_{3I}(\mathcal{R}) = \zeta_{2K}(\mathcal{R}) = 1 - (1-\zeta_{\mathrm{min}})\left(\frac{1-z_{\mathrm{max}}}{1-z_{\mathrm{min}}}\right)^{\mathcal{R}} , \tag{106}$$

for m_D:

$$\zeta_{3E}(\mathcal{R}) = \zeta_{\mathrm{min}}\left(\frac{\zeta_{\mathrm{max}}}{\zeta_{\mathrm{min}}}\right)^{\mathcal{R}} , \tag{107}$$

$$\zeta_{3F}(\mathcal{R}) = \zeta_{\mathrm{min}} + \mathcal{R}(\zeta_{\mathrm{max}} - \zeta_{\mathrm{min}}) , \tag{108}$$

$$\zeta_{3I}(\mathcal{R}) = 1 - (1-\zeta_{\mathrm{min}})\left(\frac{1-\zeta_{\mathrm{max}}}{1-\zeta_{\mathrm{min}}}\right)^{\mathcal{R}} , \tag{109}$$

for E^*:

$$\zeta_{1E}(\mathcal{R}) = \frac{\zeta_{\max}\zeta_{\min}}{\zeta_{\max} - \mathcal{R}(\zeta_{\max} - \zeta_{\min})} \,, \tag{110}$$

$$\zeta_{1F}(\mathcal{R}) = \zeta_{\min} + \mathcal{R}(\zeta_{\max} - \zeta_{\min}) \,, \tag{111}$$

for m_{g^*}:

$$\zeta_{2S}(\mathcal{R}) = \zeta_{\min} + \mathcal{R}(\zeta_{\max} - \zeta_{\min}) \,. \tag{112}$$

The generated value of ζ can now be compared to the limits imposed by the physical phase space at the generated value of Q and a rejection imposed if the generated ζ value falls outside the phase space.

A.9 Inverse Transforms

After a set of shower variables has been generated, the Q^2 and ζ choices must be inverted to reobtain the branching invariants, (s_{ij}, s_{jk}), which are used to construct the kinematics of the trial branching. These inversions are, for $Q_\perp$:

$$\begin{array}{cccccc} \mathbf{Q_\perp^2} & & \boldsymbol{\zeta_1} & \boldsymbol{\zeta_2} & \boldsymbol{\zeta_3} & \\ y_{ij} & = & \sqrt{\frac{\zeta_1}{1-\zeta_1}}\sqrt{\frac{Q_\perp^2}{N_\perp m_{IK}^2}} & \zeta_2 & \frac{1}{\zeta_3}\frac{Q_\perp^2}{N_\perp m_{IK}^2} \,, \\ y_{jk} & = & \sqrt{\frac{1-\zeta_1}{\zeta_1}}\sqrt{\frac{Q_\perp^2}{N_\perp m_{IK}^2}} & \frac{1}{\zeta_2}\frac{Q_\perp^2}{N_\perp m_{IK}^2} & \zeta_3 \,, \end{array} \tag{113}$$

for m_D (on $y_{ij} < y_{jk}$ branch):

$$\begin{array}{ccc} \mathbf{m_D^2} & & \boldsymbol{\zeta_3} \\ y_{ij} & = & \frac{m_D^2}{N_D} \,, \\ y_{jk} & = & \zeta_3 \,, \end{array} \tag{114}$$

for E^*:

$$\begin{array}{ccc} \mathbf{E^{*2}} & & \boldsymbol{\zeta_1} \\ y_{ij} & = & \zeta_1\sqrt{\frac{E^{*2}}{m_{IK}^2}} \,, \\ y_{jk} & = & (1-\zeta_1)\sqrt{\frac{E^{*2}}{m_{IK}^2}} \,, \end{array} \tag{115}$$

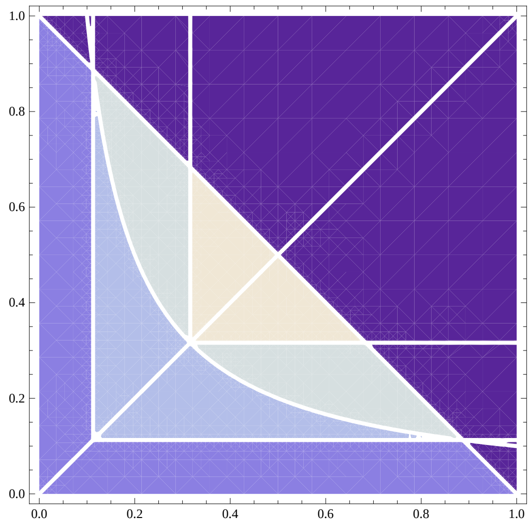

Figure 1: Illustration of the intersection/nesting of p_T and m_D contours.

for $m^2_{q\bar{q}}$:

$$\begin{aligned} \mathbf{m^2_{q\bar{q}}} & \qquad \boldsymbol{\zeta_2} \\ y_{ij} &= \zeta_2 \,, \\ y_{jk} &= \frac{m^2_{q\bar{q}}}{m^2_{IK}} \,, \end{aligned} \tag{116}$$

B Cutoff Boundaries

B.1 Fixed Transverse Momentum

- Consider the region defined by $y_{ij}y_{jk} \geq y_\perp$. For illustration, a value of $y_\perp = 0.1$ was used for the contour shown with light green shading in fig. 1.
- The (larger) invariant-mass region that completely encloses the $y_\perp$ one is defined by $y_D = \min(y_{ij}, y_{jk}) \geq \frac{1}{2}(1 - \sqrt{1 - 4y_\perp})$. This is shown with light blue shading in fig. 1.
- The (smaller) invariant-mass region that is completely enclosed by the $y_\perp$ one is defined by $y_D = \min(y_{ij}, y_{jk}) \geq \sqrt{y_\perp}$. This is shown with light yellow shading in fig. 1.

To translate this to evolution variables, with arbitrary normalization factors, use $y_\perp = Q^2_\perp/s_{IK}/N_\perp$ and $m^2_D/s_{IK}/N_D$.

B.2 Fixed Dipole Mass

- Consider the region defined by $\min(y_{ij}y_{jk}) \geq y_D$, with y_D some fixed value.

- The (larger) transverse-momentum region that completely encloses the y_D one is defined by $y_\perp = y_{ij}y_{jk} \geq y_D^2$. This relationship is illustrated by the light-green and light-yellow shaded regions in fig. 1.

- The (smaller) transverse-momentum region that is completely enclosed by the y_D one is defined by $y_\perp = y_{ij}y_{jk} \geq \frac{1}{4}(1-(1-2y_D)^2)$. This relationship is illustrated by the light-green and light-blue shaded regions in fig. 1.

To translate this to evolution variables, with arbitrary normalization factors, use $y_\perp = Q_\perp^2/s_{IK}/N_\perp$ and $m_D^2/s_{IK}/N_D$.

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